1. (2 pts) From our calculator we get: \( \sin \left( 4t \right) = \frac{1}{3} \Rightarrow 4t = \sin^{-1} \left( \frac{1}{3} \right) = 0.6435 \). From a quick sketch of a unit circle we can see that the second angle will be \( \pi - 0.6435 = 2.4981 \). Now, all solutions are,

\[
4t = 0.6435 + 2\pi n \\
4t = 2.4981 + 2\pi n
\]

\[
t = 0.1609 + \frac{\pi n}{2} \\
t = 0.6245 + \frac{\pi n}{2} \quad n = 0, \pm 1, \pm 2, \ldots
\]

Plugging in values of \( n \) gives the following solutions.

\[
n = -1: \quad t = 1.4099 < -1 \quad \text{OR} \quad t = -0.9463 \\
n = 0: \quad t = 0.1609 \quad \text{OR} \quad t = 0.6245 \\
n = 1: \quad t = 1.7317 \quad \text{OR} \quad t = 2.1953 > 2
\]

So, we have the above four solutions.

2. (2 pts) From our calculator we get: \( \cos \left( \frac{z}{4} \right) = -\frac{2}{3} \Rightarrow \frac{z}{4} = \cos^{-1} \left( -\frac{2}{3} \right) = 2.3005 \). From a quick sketch of a unit circle we can see that the second angle will be \( 2\pi - 2.3005 = 3.9827 \). Now, all solutions are,

\[
\frac{z}{4} = 2.3005 + 2\pi n \\
\frac{z}{4} = 3.9827 + 2\pi n
\]

\[
z = 9.2020 + 8\pi n \\
z = 15.9308 + 8\pi n \quad n = 0, \pm 1, \pm 2, \ldots
\]

Plugging in values of \( n \) gives the following solutions.

\[
n = 0: \quad z = 9.202 \quad \text{OR} \quad z = 15.9308 \\
n = 1: \quad z = 34.3347 \quad \text{OR} \quad z = 41.0635 > 40
\]

So, we have the above three solutions.

5. (2 pts) Not much to this one.

\[
e^{3x^2 - 7} = \frac{88}{3} \rightarrow 2x^2 - 7 = \ln \left( \frac{88}{3} \right) \rightarrow x^2 = \frac{1}{2} \left[ \ln \left( \frac{88}{3} \right) + 7 \right] \rightarrow x = \pm \sqrt[2]{\frac{1}{2} \left[ \ln \left( \frac{88}{3} \right) + 7 \right]} = \pm 2.2780
\]

7. (2 pts) First we need to combine the logarithms and then exponentiate both sides.

\[
\ln \left[ \frac{1}{2} \ln \left( \frac{88}{3} \right) + 7 \right] = -2 \rightarrow x^2 + x - 30 = e^{-2}
\]

So, we have a quadratic equation, \( x^2 + x - 30 - e^{-2} = 0 \) and we’ll need to use the quadratic equation to find the solutions.

\[
x = \frac{-1 \pm \sqrt{1 - 4(-30 - e^{-2})}}{2} = \frac{-1 \pm \sqrt{12623}}{2}, \quad 5.0123
\]

Checking in the original equation we can see that IN THIS CASE the first solution will lead to negative values in the logarithms while the second does not. Therefore the only solution is: \( x = 5.0123 \).
12. (2 pts) Here is the table of values.

<table>
<thead>
<tr>
<th>t</th>
<th>f(t)</th>
<th>t</th>
<th>f(t)</th>
</tr>
</thead>
<tbody>
<tr>
<td>-1.9</td>
<td>1.261088968</td>
<td>-2.1</td>
<td>0.8040974487</td>
</tr>
<tr>
<td>-1.99</td>
<td>1.022826421</td>
<td>-2.01</td>
<td>0.9778194725</td>
</tr>
<tr>
<td>-1.999</td>
<td>1.002253233</td>
<td>-2.001</td>
<td>0.9977532257</td>
</tr>
<tr>
<td>-1.9999</td>
<td>1.000225032</td>
<td>-2.0001</td>
<td>0.9997750323</td>
</tr>
</tbody>
</table>

So, from the values of this table it looks like we can estimate,

$$\lim_{t \to -2} \frac{1 - e^{4t+8}}{t^2 - 4} = 1$$

---

**Not Graded**

3. Solving for cosine gives: $\cos(8x) = 4$. Now, we know that $-1 \leq \cos \theta \leq 1$ and so it is not possible for cosine to ever equal 4. Therefore, there are no solutions to this equation.

4. From our calculator we get: $\sin\left(\frac{\pi}{2}\right) = -\frac{3}{5} \implies \frac{\pi}{2} = \sin^{-1}\left(-\frac{3}{5}\right) = -0.6435$. From a quick sketch of a unit circle we can see that a positive angle corresponding to this is $2\pi - 0.6435 = 5.6397$. You don’t need to use this but I will for these solutions. We can also see from the unit circle that the second angle will be $\pi + 0.6435 = 3.7851$. Now, all solutions are,

$$\frac{\pi}{2} = 3.7851 + 2\pi n \quad \Rightarrow \quad y = 7.5702 + 4\pi n \quad n = 0, \pm 1, \pm 2, \ldots$$

$$\frac{\pi}{2} = 5.6397 + 2\pi n \quad \Rightarrow \quad y = 11.2794 + 4\pi n$$

Plugging in values of $n$ gives the following solutions.

- $n = 0$: $y = 7.5702 < 20$ OR $y = 11.2794 < 20$
- $n = 1$: $y = 20.1366$ OR $y = 23.8458$
- $n = 2$: $y = 32.7030$ OR $y = 36.4122 > 35$

So, we have the above three solutions.

6. Simplify the equation and don’t forget you can’t cancel terms from both sides of the equation when solving unless you KNOW that they won’t be zero.

$$2x - 3 - 2(2x - 3)e^{5x-12} = 0 \quad \Rightarrow \quad (2x - 3)(1 - 2e^{5x-12}) = 0$$

From this we can see that we have two solutions: $x = \frac{3}{2}$ and

$$1 - 2e^{5x-12} = 0 \quad \Rightarrow \quad e^{5x-12} = \frac{1}{2} \quad \Rightarrow \quad 5x - 12 = \ln\left(\frac{1}{2}\right) \quad \Rightarrow \quad x = \frac{1}{5}\left(12 + \ln\left(\frac{1}{2}\right)\right) = 2.2614$$
8. The average velocity is,

\[
AV = \frac{s(t) - s(0)}{t-0} = \frac{\ln(8t + 4) - t^2 + 6t - \ln(4)}{t}
\]

Here is the table of values for the given values of \( t \).

<table>
<thead>
<tr>
<th>( t )</th>
<th>( AV )</th>
<th>( t )</th>
<th>( AV )</th>
</tr>
</thead>
<tbody>
<tr>
<td>-0.1</td>
<td>8.331435513</td>
<td>0.1</td>
<td>7.723215568</td>
</tr>
<tr>
<td>-0.01</td>
<td>8.030270732</td>
<td>0.01</td>
<td>7.97026273</td>
</tr>
<tr>
<td>-0.001</td>
<td>8.003002671</td>
<td>0.001</td>
<td>7.997002663</td>
</tr>
<tr>
<td>-0.0001</td>
<td>8.000300027</td>
<td>0.0001</td>
<td>7.999700027</td>
</tr>
</tbody>
</table>

From the table it looks like we can estimate that the velocity at \( t = 0 \) is 8 and so the object is moving to the right.

9. The slopes of the secant lines are,

\[
m_{PQ} = \frac{f(x) - f(3)}{x-3} = \frac{\sin(x^2 - 9) - \sin(0)}{x-3} = \frac{\sin(x^2 - 9)}{x-3}
\]

The slopes are then,

<table>
<thead>
<tr>
<th>( x )</th>
<th>( m_{PQ} )</th>
<th>( x )</th>
<th>( m_{PQ} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>2.9</td>
<td>5.563610229</td>
<td>3.1</td>
<td>5.728674601</td>
</tr>
<tr>
<td>2.99</td>
<td>5.986418613</td>
<td>3.01</td>
<td>6.006382623</td>
</tr>
<tr>
<td>2.999</td>
<td>5.998964018</td>
<td>3.001</td>
<td>6.000963982</td>
</tr>
<tr>
<td>2.9999</td>
<td>5.999899964</td>
<td>3.0001</td>
<td>6.00009964</td>
</tr>
</tbody>
</table>

It looks like the tangents of the secant lines are moving towards 6 and so we can estimate that the slope is \( m = 6 \). The tangent line is then,

\[
y = f(3) + m(x - 3) = \sin(0) + 6(x - 3) = 6(x - 3)
\]

10. To say \( \lim_{x \to 7} f(x) = -25 \) we mean that as we let \( x \) approach the value of 7, from both the left and the right, the function, \( f(x) \), is getting closer and closer to the value of -25. Also, it is completely possible to have \( f(7) = 100 \) because we know that the limit does not care about the that function is doing at \( x = 7 \) and it only cares about what the function is doing around the point. Therefore the function does NOT have to have the same value as the limit.
11. Here is the table of values.

<table>
<thead>
<tr>
<th>x</th>
<th>f(x)</th>
<th>x</th>
<th>f(x)</th>
</tr>
</thead>
<tbody>
<tr>
<td>3.9</td>
<td>-1.256410256</td>
<td>4.1</td>
<td>-1.243902439</td>
</tr>
<tr>
<td>3.99</td>
<td>-1.250626566</td>
<td>4.01</td>
<td>-1.249376559</td>
</tr>
<tr>
<td>3.999</td>
<td>-1.250062516</td>
<td>4.001</td>
<td>-1.249937516</td>
</tr>
<tr>
<td>3.9999</td>
<td>-1.25000625</td>
<td>4.0001</td>
<td>-1.24999375</td>
</tr>
</tbody>
</table>

So, from the values of this table it looks like we can estimate,

\[
\lim_{x \to 4} \frac{x^2 - 3x - 4}{4x - x^2} = -1.25
\]