1. (2 pts)
(a) 
\[ f(-1) = 2 \quad \lim_{x \to -1} f(x) = 2 \quad \lim_{x \to -1} f(x) = 1 \]
\[ \lim_{x \to -1} f(x) \text{ doesn't exist } b/c \quad \lim_{x \to -1} f(x) \neq \lim_{x \to -1} f(x) \]
(b) 
\[ f(2) = -3 \quad \lim_{x \to 2} f(x) = -1 \quad \lim_{x \to 2} f(x) = -1 \quad \lim_{x \to 2} f(x) = -1 \]
(c) 
\[ f(3) \text{ doesn't exist } \]
\[ \lim_{x \to 3} f(x) = 4 \quad \lim_{x \to 3} f(x) = 4 \quad \lim_{x \to 3} f(x) = 4 \]

3. (2 pts) 
\[ \lim_{x \to 4} \frac{4 + 7x - 2x^2}{x^2 - 16} = \lim_{x \to 4} \frac{-(x - 4)(2x + 1)}{(x - 4)(x + 4)} = \lim_{x \to 4} \frac{2x + 1}{x + 4} = \frac{9}{8} \]

5. (2 pts) 
\[ \lim_{y \to 1} \frac{3y - \sqrt{y^2 + 8}}{y^2 + 2} = \lim_{y \to 1} \frac{3y + \sqrt{y^2 + 8}}{(y^2 + 2)} = \lim_{y \to 1} \frac{9y^2 - (y^2 + 8)}{3y + \sqrt{y^2 + 8}} \]
\[ = \lim_{y \to 1} \frac{8(y - 1)(y + 1)}{(y + 2)(y - 1)(3y + \sqrt{y^2 + 8})} \]
\[ = \lim_{y \to 1} \frac{8(y + 1)}{(y + 2)(3y + \sqrt{y^2 + 8})} = \frac{8}{9} \]

6. (a) NOT GRADED!! In this case we can use the second formula because \( x = 12 > -3 \) is completely inside this region.
\[ \lim_{x \to 12} g(x) = \lim_{x \to 12} (x + 7) = 19 \]
(b) (2 pts) – ONLY GRADED THIS PART!! Here we will need to look at the two one-sided limits because \( x = -3 \) is the “cut-off” point.
\[ \lim_{x \to -3^-} g(x) = \lim_{x \to -3^-} e^{2-x} = e^5 \]
\[ \lim_{x \to -3^+} g(x) = \lim_{x \to -3^+} (x + 7) = 4 \]
\[ \lim_{x \to -3^+} g(x) \neq \lim_{x \to -3^-} g(x) \]
The two one-sided limits are not the same and so \( \lim_{x \to -3} g(x) \) doesn’t exist.

8. (2 pts) In both of these limits the numerator is staying fixed at 9 and as \( x \) approaches 3 (from either side) we can see that \( 6-2x \) is approaching zero. So, we have a fixed number divided by something increasingly smaller and so it should make some sense that both of these are going to either \( \infty \) or \( -\infty \) and this will depend upon the sign of the denominator.
In the first case $6-2x$ is positive because $x < 3$ and raising this to the $5^{th}$ power will not change this. So, in this case we have a fixed positive number in the numerator divided by something increasingly smaller and positive and so the limit in this case will be $\infty$.

In the second case $6-2x$ is negative because $x > 3$ and raising this to the $5^{th}$ power will not change this. So, in this case we have a fixed positive number in the numerator divided by an increasingly smaller and negative number and so the limit in this case will be $-\infty$.

Because the two one-sided limits are not the same the overall limit does not exist. Here are the official answers to this problem.

\[
\lim_{x \to 3^-} \frac{9}{(6-2x)^5} = \infty \quad \lim_{x \to 3^+} \frac{9}{(6-2x)^5} = -\infty \quad \lim_{x \to 3} \frac{9}{(6-2x)^5} \text{ doesn't exist}
\]

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**Not Graded**

2. I believe I used the same property numbers in class as in my notes online, but just in case the properties listed here are those from the notes online.

(a) \[
\lim_{x \to 12} \left[ f(x) + 7g(x) - 9 \right] = \lim_{x \to 12} f(x) + \lim_{x \to 12} 7g(x) - \lim_{x \to 12} 2 \quad \text{Property 2}
\]
\[
= \lim_{x \to 12} f(x) + 7 \lim_{x \to 12} g(x) - \lim_{x \to 12} 2 \quad \text{Property 1}
\]
\[
= \lim_{x \to 12} f(x) + 7 \lim_{x \to 12} g(x) - 2 \quad \text{Property 7}
\]
\[
= 9 + 7(-1) - 2 = [0] \quad \text{Plug in values of limits}
\]

(b) \[
\lim_{x \to 12} \frac{g(x)}{\left[ f(x) \right]^2 - 5h(x)} = \frac{\lim_{x \to 12} g(x)}{\lim_{x \to 12} \left[ \left( f(x) \right)^2 - 5h(x) \right]} \quad \text{Property 4}
\]
\[
= \frac{\lim_{x \to 12} g(x)}{\lim_{x \to 12} \left[ f(x) \right]^2 - \lim_{x \to 12} 5h(x)} \quad \text{Property 2}
\]
\[
= \frac{\lim_{x \to 12} g(x)}{\lim_{x \to 12} \left[ f(x) \right]^2 - 5 \lim_{x \to 12} h(x)} \quad \text{Property 5 & 1}
\]
\[
= \frac{-1}{[9]^2 - 5(-4)} = \frac{-1}{101} \quad \text{Plug in values of limits}
\]
4. \[ \lim_{{t \to -3}} \frac{(t+1)(t+5)+3t+13}{t^2+2t-3} = \lim_{{t \to -3}} \frac{t^2+9t+18}{t^2+2t-3} = \lim_{{t \to -3}} \frac{(t+3)(t+6)}{(t+3)(t-1)} = \lim_{{t \to -3}} \frac{t+6}{t-1} = -\frac{3}{4} \]

7. Note that we can’t just cancel the \( h \)’s since once is inside the absolute value bars. So, we’ll use the hint and recall that,

\[ |h| = \begin{cases} h & \text{if } h \geq 0 \\ -h & \text{if } h < 0 \end{cases} \]

With this we can do the two one sided limits to eliminate the absolute value bars.

\[ \lim_{{h \to 0^+}} \frac{|h|}{h} = \lim_{{h \to 0^+}} \frac{-h}{h} = \lim_{{h \to 0^+}} -1 = -1 \quad \text{because } h < 0 \text{ in this case} \]

\[ \lim_{{h \to 0^-}} \frac{|h|}{h} = \lim_{{h \to 0^-}} \frac{h}{h} = \lim_{{h \to 0^-}} 1 = 1 \quad \text{because } h > 0 \text{ in this case} \]

The two one-sided limits are not the same and so \( \lim_{{h \to 0}} \frac{|h|}{h} \) does not exist.

9. In both of these limits the numerator is staying fixed at -2 and as \( w \) approaches -9 (from either side) we can see that \( w+9 \) is approaching zero. So, we have a fixed number divided by something increasingly smaller and so it should make some sense that both of these are going to either \( \infty \) or \( -\infty \) and this will depend upon the sign of the denominator

In the first case \( w+9 \) is negative because \( w < -9 \) and raising this to the 10th power will make this positive. So, in this case we have a fixed negative number in the numerator divided by something increasingly smaller and positive and so the limit in this case will be \( -\infty \).

In the second case \( w+9 \) is positive because \( w > -9 \) and raising this to the 10th power will not change this. So, in this case we have a fixed negative number in the numerator divided by an increasingly smaller and positive number and so the limit in this case will be \( -\infty \).

Alternatively instead of the previous two paragraphs we could just acknowledge that the exponent in the denominator is even and so the denominator will be positive regardless of the sign of \( w+9 \).

Because the two one-sided limits are the same the overall limit will also be \( -\infty \). Here are the official answers to this problem.

\[ \lim_{{w \to -9}} \frac{-2}{(w+9)^{10}} = -\infty \quad \lim_{{w \to -9}} \frac{-2}{(w+9)^{10}} = -\infty \quad \lim_{{w \to -9}} \frac{-2}{(w+9)^{10}} = -\infty \]