2. (2 pts)
\[ \lim_{z \to \infty} \frac{(2 + 3z)(4z - 1)}{4 - z^2} = \lim_{z \to \infty} \frac{12z^2 + 5z - 2}{4 - z^2} = \lim_{z \to \infty} \frac{z^2 \left(12 + \frac{5}{z} - \frac{2}{z^2}\right)}{4 - \frac{1}{z^2}} = \lim_{z \to \infty} \frac{12 + \frac{5}{z} - \frac{2}{z^2}}{4} = \frac{12}{1} = 12 = -12 \]

4. (2 pts)
\[ \lim_{y \to \infty} \frac{8 - 4y}{\sqrt{7y^2 - 1}} = \lim_{y \to \infty} \frac{y \left(\frac{8}{y} - 4\right)}{\sqrt{7 - \frac{1}{y}}} = \lim_{y \to \infty} \frac{y \left(\frac{8}{y} - 4\right)}{y \sqrt{7 - \frac{1}{y}}} = \frac{\frac{8}{y} - 4}{\sqrt{7 - \frac{1}{y}}} = \frac{-4}{\sqrt{7}} \]
Assume \( y > 0 \) because \( y \to \infty \) so \( |y| = y \)

For the second limit the work will be identical until we get rid of the absolute value. In this case, we can assume that \( y < 0 \) because \( y \to -\infty \) and so \( |y| = -y \). So, picking up we get,
\[ \lim_{y \to \infty} \frac{8 - 4y}{\sqrt{7y^2 - 1}} = \lim_{y \to \infty} \frac{y \left(\frac{8}{y} - 4\right)}{\sqrt{7 - \frac{1}{y}}} = \lim_{y \to \infty} \frac{\frac{8}{y} - 4}{\sqrt{7 - \frac{1}{y}}} = \frac{4}{\sqrt{7}} \]

6. (2 pts) Not much to do here. The function is continuous because it is a difference/product of continuous functions. Now all we need to do is evaluate the function at the two points.
\[ f(-15) = 11.7923 \quad f(-7) = -4.0547 \]
So, we can see that \( f(-7) = -4.0547 < 0 < f(-15) = 11.7923 \) and so, by the IVT there is a number \( c \) such that \( -15 < c < -7 \) such that \( f(c) = 0 \). Or in other words, \( c \) is a root of the function. For the sake of completeness we can use a computer to see that \( x = -11.48494037 \) is a root of the function.

9. (2 pts)
\[ f''(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h} = \lim_{h \to 0} \frac{7 + 2(x+h) - 3(x+h)^2 - (7 + 2x - 3x^2)}{h} \]
\[ = \lim_{h \to 0} \frac{7 + 2x + 2h - 3x^2 - 6xh - 2h^2 - 7 - 2x + 3x^2}{h} \]
\[ = \lim_{h \to 0} \frac{2h - 6xh - 2h^2}{h} = \lim_{h \to 0} (2 - 6x - 2h) = 2 - 6x \quad \Rightarrow \quad f''(x) = 2 - 6x \]

12. (2 pts) Not much to do here other than compute the value of the derivative at the points in question.
\[ g'(-8) = -5 < 0 \quad \text{decreasing} \quad g'(3) = -5 < 0 \quad \text{decreasing} \]
Note that, in this case, we didn’t really need to do any evaluation to answer this. The derivative is a constant -5 and so will always be negative and hence the function will always be decreasing.

Not Graded

1. \[ \lim_{t \to \infty} \frac{10t^2 + 8t}{7 - 6t + 2t^3} = \lim_{t \to \infty} t^3 \left( \frac{10}{t} + \frac{8}{t^2} \right) = \lim_{t \to \infty} \frac{10}{t} + \frac{8}{t^2} = [0] \]

3. \[ \lim_{p \to -\infty} \frac{9 + 5p^4}{5p^3 + 9p^2 - 1} = \lim_{p \to -\infty} \frac{p^3 \left( \frac{9}{p^4} + 5 \right)}{p^3 \left( 5 + \frac{9}{p} - \frac{1}{p^2} \right)} = \lim_{p \to -\infty} \frac{9}{p^4} + 5 = -\infty \]

5. In this case we need to determine where the denominator will be zero. So, all we need to do is set the denominator equal to zero and solve.
\[ 2x^2 e^{3-x} - x^2 e^{2x+9} = x^2 \left( 2e^{3-x} - e^{2x+9} \right) = 0 \]

So, we get \( x = 0 \) as one point and we’ll need to solve the second equation.
\[ 2e^{3-x} - e^{2x+9} = 0 \rightarrow 2e^{3-x} = e^{2x+9} \rightarrow 2 = \frac{e^{2x+9}}{e^{3-x}} = e^{3x-6} \rightarrow 3x - 6 = \ln(2) \rightarrow x = \frac{1}{3} \left( 6 + \ln(2) \right) \]

The function will therefore not be continuous at \( x = 0 \) at \( x = \frac{1}{3} \left( 6 + \ln(2) \right) = 2.2310 \).

7. This problem seems quite complicated at first glance, but it isn’t as bad as it might seem. We’re being asked to find an interval with a width of no more than \( \frac{1}{2} \) in which the function will have a value of 8 in \([0,5]\). Let’s just need to start by evaluating the function at all the integers in this interval.
\[ A(0) = 0 \quad A(1) = 2.483 \quad A(2) = 0.938 \quad A(3) = 2.269 \quad A(4) = 10.758 \quad A(5) = 2.902 \]

So, from these numbers the IVT tells us that the function will have a value of 8 in the intervals \([3,4]\) and \([4,5]\). These are both intervals of width 1 and so not quite what we want. However, with a couple of further computations we can get what we’re looking for.
\[ A(3.5) = 6.751 \quad A(4.5) = 6.795 \]

The IVT now tells us that the function will have a value of 8 in the intervals \([3.5, 4]\) and \([4, 4.5]\). Either of these will work as answers for this problem.

8. \[ g'(x) = \lim_{h \to 0} \frac{g(x+h) - g(x)}{h} = \lim_{h \to 0} \frac{2 - 5(x+h) - (2 - 5x)}{h} = \lim_{h \to 0} \frac{-5h}{h} = \lim_{h \to 0} -5 = -5 \]
10. \[ Q'(t) = \lim_{h \to 0} \frac{O(t+h)-O(t)}{h} = \lim_{h \to 0} \frac{\sqrt[4]{4(t+h)+1} - \sqrt[4]{4t+1}}{h} \frac{\sqrt[4]{4(t+h)+1} + \sqrt[4]{4t+1}}{\sqrt[4]{4(t+h)+1} + \sqrt[4]{4t+1}} \] 
\[ = \lim_{h \to 0} \frac{4t+4h+1-(4t+1)}{h \left( \sqrt[4]{4(t+h)+1} + \sqrt[4]{4t+1} \right)} = \lim_{h \to 0} \frac{4h}{h \left( \sqrt[4]{4(t+h)+1} + \sqrt[4]{4t+1} \right)} \] 
\[ = \lim_{h \to 0} \frac{4}{\sqrt[4]{4(t+h)+1} + \sqrt[4]{4t+1}} = \frac{4}{2 \sqrt[4]{4t+1}} = \frac{2}{\sqrt[4]{4t+1}} \quad \Rightarrow \quad Q'(t) = \frac{2}{\sqrt[4]{4t+1}} \]

11. This is not as tricky as it might seem at first glance. Let’s start with the definition of the derivative.
\[ f''(x) = \lim_{h \to 0} \frac{f(x+h)-f(x)}{h} = \lim_{h \to 0} \frac{1}{h} \left( 4(x+h) - \frac{1}{x+h} \left( 4x - \frac{1}{x} \right) \right) \]

Now, to make this “easier” all we need to do is split this up into two limits as follows,
\[ f''(x) = \lim_{h \to 0} \frac{1}{h} \left( 4(x+h) - \frac{1}{x+h} \right) + \lim_{h \to 0} \frac{1}{h} \left( - \frac{1}{x+h} - \frac{1}{x} \right) = \lim_{h \to 0} \frac{1}{h} \left( 4h \right) + \lim_{h \to 0} \frac{1}{h} \left( - \frac{x+(x+h)}{x(x+h)} \right) \]
\[ = \lim_{h \to 0} \frac{4}{h} + \lim_{h \to 0} \frac{1}{h} \left( \frac{h}{x(x+h)} \right) = \lim_{h \to 0} \frac{4}{h} + \lim_{h \to 0} \frac{1}{h} \left( \frac{1}{x(x+h)} \right) = 4 + \frac{1}{x^2} \quad \Rightarrow \quad f''(x) = 4 + \frac{1}{x^2} \]

13. All we need here is: \( f'(7) = \sqrt{29} \) \quad \& \quad f''(7) = \frac{2}{\sqrt{29}}. \) The tangent line is then,
\[ y = f'(7)(x-7) + f(7) = \sqrt{29} + \frac{2}{\sqrt{29}}(x-7) \]

14. We know that the function will stop changing if \( f''(x) = 0 \) so all we need to solve is,
\[ 0 = f''(x) = 2 - 6x \quad \Rightarrow \quad x = \frac{1}{3} \]

So, the function will stop changing at \( x = \frac{1}{3} \).

15. The function will stop changing at \( f'(x) = 0 \) and from our graph we can see that this will occur at the points \( x = -3, 1, 6 \). Also the function will be increasing if \( f''(x) > 0 \) and decreasing if \( f''(x) < 0 \). So, again from our graph we see that,
Increasing: \( x < -3, \ x > 6 \) \quad \text{Decreasing:} \quad -3 < x < 1, \ 1 < x < 6