1. (2 pts)

\[
g'(z) = 3(8z)(4z^2-1)^2(6-3z)^4 + (4)(-3)(4z^2-1)^3(6-3z)^3 \\
= -12(4z^2-1)^2(6-3z)^3[-2z(6-3z)+(4z^2-1)] \\
= -3(4z^2-1)^2(6-3z)^3[10z^2-12z-1]
\]

The five critical points are then: \( z = \pm \frac{1}{2}, \ z = 2, \ z = \frac{6z\sqrt{46}}{16} = -0.0782, 1.2782 \).

5. (2 pts) \( W'(y) = \frac{3}{3y+6} - \frac{8y}{y^2+6} = \frac{3(y^2+6) - 8y(3y+6)}{(3y+6)(y^2+6)} = \frac{-3(7y^2+16y-6)}{(3y+6)(y^2+6)} \)

Critical points are then where the derivative is zero (i.e. where the numerator is zero) and where the derivative doesn’t exist (i.e. where the denominator is zero). For the critical points we need to recall that the function also has to exist at these points. The argument of the second logarithm is always positive and so we don’t need to worry about that one. However, the first logarithm will only exist if \( y > -2 \) and so we’ll need to keep this in mind as we do our critical point work.

From the numerator we get the two possible critical points: \( y = \pm \left( -8 \pm \sqrt{16} \right) = \pm 2.6137, 0.3279 \).

From the denominator we get the no critical points: \( t = \infty \).

Note that if \( y = -2.6137 \) the first logarithm in the function will have a negative argument and so not exist. Therefore, this point can’t be a critical point. Also, if \( y = -2 \) the argument of the first logarithm is zero and this also can’t be a critical point. Therefore there is a single critical point for this function: \( y = 0.3279 \).

7. (2 pts) From \#1 we know that the critical points are: \( z = \pm \frac{1}{2}, \ z = 2, \ z = \frac{6z\sqrt{46}}{16} = -0.0782, 1.2782 \).

However only \( z = \pm \frac{1}{2}, \ z = \frac{6z\sqrt{46}}{16} = -0.0782 \) are in the given interval. So evaluating the function at these critical points and the endpoints of the interval gives,

\[
g\left( \frac{-1}{2} \right) = 0 \quad g\left( \frac{1}{2} \right) = 0 \quad g\left( -0.0782 \right) = -1402.7115 \quad g\left( -1 \right) = 177,147 \quad g\left( 1 \right) = 2187
\]

So the absolute max is 177,147 which occurs at \( z = -1 \) and the absolute minimum is -1402.7115 which occurs at \( z = \frac{6z\sqrt{46}}{16} = -0.0782 \).
9. (2 pts) We have done all the work for this problem already. We are trying to determine if 
\( P(t) > 200 \) (recall the function is in hundreds...) in the interval \([0,8]\) and from #8 we found that the 
absolute maximum of the function in \([0,8]\) was 168.7462 and so the population will clearly NOT be 
above 20,000.

12. (2 pts) The first derivative is,
\[
\frac{d^4}{dx^4} f(x) = 15x^4 - 20x^3 - 180x^2 = 5x^2(3x^2 - 4x - 36)
\]
The critical points are then: \( x = 0, x = \frac{2 \pm \sqrt{4^2 - 4 \cdot 3 \cdot 36}}{2} = -2.861, 0, 4.1943 \). Here are the values of the 
derivative I used for my number line.
\[
\begin{align*}
g'(-3) &= 135 \\
g'(-1) &= -145 \\
g'(1) &= -185 \\
g'(5) &= 2375
\end{align*}
\]
From this we get the following increasing/decreasing information.

- Increasing: \( -\infty < x < -2.861, \ 4.1943 < x < \infty \)
- Decreasing: \( -2.861 < x < 0, \ 0 < x < 4.1943 \)

We also have the following classifications of the critical points.

- \( x = -2.861 \): Relative Maximum
- \( x = 0 \): Neither
- \( x = 4.1943 \): Relative Minimum

---

**Not Graded**

2.
\[
f''(x) = 8 + 8 \cos \left(\frac{x}{2}\right) \Rightarrow \cos \left(\frac{x}{2}\right) = -1 \Rightarrow \frac{x}{2} = \pi + 2\pi n \quad \Rightarrow \quad x = 2\pi + 4\pi n, \ n = 0, \pm 1, \pm 2, \ldots
\]
Note that this was one of those rare cases where there really was only a single angle that satisfies the 
equation between 0 and 2\( \pi \).

3. \( h'(x) = 30x^5 - 60x^4 - 240x^3 = 30x^3(x + 2)(x - 4) \quad \Rightarrow \quad x = -2, 0, 4 \)

4.
\[
P'(t) = 2t(3t - 21)^{3/2} + (1 + t^2)(3t - 21)^{3/2} = \frac{2t(3t - 21) + 1 + t^2}{(3t - 21)^{3/2}} = \frac{7t^2 - 42t + 1}{(3t - 21)^{3/2}}
\]
Critical points are then where the derivative is zero (\( i.e. \) where the numerator is zero) and where the 
derivative doesn’t exist (\( i.e. \) where the denominator is zero). For the critical points we need to recall 
that the function also has to exist at these points (this is especially true for the second set in this case).

From the numerator we get the two critical points: \( t = \frac{1}{3} \left(21 \pm \sqrt{434}\right) = 0.02390, \ 5.9761 \).
From the denominator we get the critical point: \( t = 7 \) (function exists at this point!).

6. Rel. Max: \( b, d, f \)       Rel. Min.: \( c, e \)       Abs. Max.: \( d \)       Abs. Min.: \( g \)

8. From #4 we know that the critical points are: \( t = 0.0239, 5.9761, 7 \). These are all in the given interval. So evaluating the function at these three critical points and the endpoints of the interval gives,

\[
P(0.0239) = 72.2426 \quad P(5.9761) = 21.6311 \quad P(0) = 72.2411 \quad P(8) = 168.7462
\]

So the absolute max is 168.7462 which occurs at \( t = 8 \) and the absolute minimum is 72.2411 which occurs at \( t = 0 \). Note that the value at \( t = 0.0239 \) is VERY close the minimum value but isn’t. Be careful with these kinds of situations and don’t round too much or you may miss things like this.

10. We’ll need to do all the work for this one. Here are the derivatives and critical points.

\[
V''(t) = -(6t^2 - 12t + 9)e^{2t^3 - 6t^2 + 9t} = -3(t-1)(t-3)e^{2t^3 - 6t^2 + 9t} \Rightarrow t = 1, t = 3
\]

Only the first critical point is in the interval \([0, 2]\) and so to find the absolute minimum (which will allow us to answer the question) we simply need to evaluate the function at the critical point and the endpoints.

\[
V(1) = 5.4018 \quad V(0) = 59 \quad V(2) = 52.6109
\]

The absolute minimum is 5.4018 and so the amount of coolant does go below 10 gallons and the machine will get shut down. For the sake of completeness we can also note that the absolute maximum was 59 even though we didn’t need that for this problem.

11. From #2 the critical points are \( x = 2\pi + 4\pi n, \ n = 0, \pm 1, \pm 2, \ldots \) and by plugging in \( n \)'s we can see that the only ones that fall in the interval are \( x = -2\pi, 2\pi, 6\pi = -6.2832, \ 6.2832, \ 18.8496 \). Here are the values of the derivative I used for my number line.

\[
f'(-10) = 10.2693 \quad f'(0) = 16 \quad f'(20) = 1.2874
\]

So, we can see that function will be increasing everywhere in the interval except at the critical points themselves where the function won’t be changing. This also means that the critical points are neither relative minimums or relative maximums.