

The Shape of a Graph, Part II

1. Determine the intervals where $h(x) = x^6 + 3x^5 - 20x^4 - 3x + 14$ is concave up and concave down.

What are the inflection points for this function?

2. Use $f(x) = 3x^5 - 5x^4 - 60x^3$ for this problem.

(a) What are the intervals of increase/decrease for this function?

(b) What are the relative extrema of this function?

(c) Find the intervals of concave up/concave down for this function.

(d) Find all the inflection points for this function.

(e) Use the information above to sketch the graph of this function.

3. Suppose that you know that the critical points of $f(x)$ are $x = -1$, $x = 2$, and $x = 4$ and that the second derivative of $f''(x) = 4x^3 - 21x^2 + 24x + 4$. If possible, classify the critical points. If it is not possible clearly explain why it is not possible.

The Mean Value Theorem

4. Verify that $g(x) = 5x + \ln(2 - x^2) - 3$ satisfies the conditions of the Mean Value Theorem on $[0, 1]$ and find all values of c that satisfy the conclusion of the Mean Value Theorem on $[0, 1]$.

5. Suppose that we know that $f(x)$ is a continuous and differentiable function and that $f(x)$ has roots at $x = a$ and at $x = b$. Show that $f(x)$ must have a critical point in the interval $[a, b]$.

Optimization

In order to receive any credit for problems 6 – 8 you MUST use Calculus techniques to find the answer. Any decimal work should include at least 4 decimal places.

6. We are going to fence in a field and the fence must enclose 100 m^2 . If the material for the “front” portion of the fence costs $\$15/\text{m}$ and the material for the remaining sides costs $\$18/\text{m}$ what are the dimensions of the field that will minimize the cost?

7. Find the point(s) on $\frac{x^2}{4} + \frac{y^2}{36} = 1$ that are closest to the point $(0, -2)$.

8. We have 100 cm^2 of material to build a cylindrical can that will include the sides and bottom, but NOT the top. What are the dimensions of the can that will give the maximum enclosed volume?

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L'Hospitals Rule

Evaluate each of the following limits.

9. $\lim_{t \rightarrow -\infty} \frac{5t^2 + 6}{3t - 7t^2}$

10. $\lim_{t \rightarrow -2} \frac{9 + 4t - e^{4t+8}}{t^3 + 4t^2 + 4t}$

11. $\lim_{w \rightarrow \infty} \frac{4 - e^{5w}}{2w + 8e^{5w}}$

12. $\lim_{x \rightarrow \infty} \left[3x \ln \left(1 - \frac{2}{x} \right) \right]$

Differentials

Compute the differential for each of the following functions.

13. $u = z \cos(3z)$

14. $u = \sec(1 + e^{-x^2})$