1. (3 pts) The derivatives are,

\[
h'(x) = 6x^5 + 15x^4 - 80x^3 - 3
\]

\[
h''(x) = 30x^4 + 60x^3 - 240x^2 = 30x^2(x + 4)(x - 2)
\]

So, it looks like possible inflection points are: \(x = -4, 0, 2\). Here are the values of the second derivative I used for my number line.

\[
h''(-5) = 5250 \quad h''(-1) = -270 \quad h''(1) = -150 \quad h''(3) = 1890
\]

From this we get the following concavity information.

- Concave Up: \(-\infty < x < -4, 2 < x < \infty\)
- Concave Down: \(-4 < x < 0, 0 < x < 2\)

The inflection points are then: \(x = -4, 2\).

8. (3 pts) A quick sketch is to the right. Here are the equations we need.

Maximize: \(V = \pi r^2 h\)  
Constraint: \(100 = 2\pi rh + \pi r^2\)

Solve the constraint for \(h\) and plug into the volume equation.

\[
h = \frac{1}{2\pi r}(100 - \pi r^2)
\]

\[
V(r) = \pi r^2 \left[ \frac{1}{2\pi r}(100 - \pi r^2) \right] = 50r - \frac{1}{2}\pi r^3
\]

\[
V'(r) = 50 - \frac{3}{2}\pi r^2 \quad V''(r) = -3\pi r
\]

From the first derivative we get the critical points: \(r = \pm\sqrt{\frac{100}{3\pi}} = \pm 3.2574\).

Because we are working with a physical dimension here only the positive makes sense.

From the second derivative we can see that provided \(r\) is positive the second derivative will be negative and so the volume will always be concave down and so the single critical point must be an absolute maximum. The dimensions are then: \(r = 3.2574, h = \frac{1}{2\pi(3.2574)}(100 - \pi(3.2574)^2) = 3.2573\)

9. (2 pts) \(\lim_{t \to -\infty} \frac{5t^2 + 6}{3t - 7t^2} = \lim_{t \to -\infty} \frac{10t}{3 - 14t} = \lim_{t \to -\infty} \frac{10}{-14} = \frac{-5}{7}\)

11. (2 pts) \(\lim_{w \to \infty} \frac{4 - e^{5w}}{2w + 8e^{5w}} = \lim_{w \to \infty} \frac{-5e^{5w}}{2 + 40e^{5w}} = \lim_{w \to \infty} \frac{-25e^{5w}}{200e^{5w}} = \lim_{w \to \infty} \frac{-1}{8} = \frac{-1}{8}\)
2. The derivatives are,

\[ f'(x) = 15x^4 - 20x^3 - 180x^2 = 5x^2(3x^2 - 4x - 36) \]
\[ f''(x) = 60x^3 - 60x^2 - 360x = 60x(x-3)(x+2) \]

From #12 of the previous homework set we have,

| Increasing | Increasing: \(-\infty < x < -2.861, \ 4.1943 < x < \infty\) |
| Decreasing: | Decreasing: \(-2.861 < x < 0, \ 0 < x < 4.1943\) |

\[ x = -2.861: \text{Relative Maximum} \quad x = 0: \text{Neither} \quad x = 4.1943: \text{Relative Minimum} \]

Now, from the second derivative the possible inflection points are

\[ x = -2, 0, 3 \]

Here are the values of the second derivative that I used for my number line.

\[ f''(-3) = -1080 \quad f''(-1) = 240 \quad f''(1) = -360 \quad f''(4) = 2375 \]

From this we have the following concavity information.

| CU: | CU: \(-2 < x < 0, \ 3 < x < \infty\) |
| CD: | CD: \(-\infty < x < -2, \ 0 < x < 3\) |

The inflections points are then,

\[ x = -2, 0, 3 \]

Finally, a quick sketch of the graph is to the right.

3. This isn’t as bad as it might seem. All we need (and in fact all we can do) to do is to use the second derivative test.

\[ f''(-5) = -45 \quad \text{Relative Maximum} \]
\[ f''(2) = 0 \quad \text{Don’t Know} \]
\[ f''(4) = 20 \quad \text{Relative Minimum} \]

Note that with the second one we can’t say what it is without the first derivative. A second derivative of zero can be anything and so we just do not know from the information provided.
4. This is a polynomial and so is continuous and differentiable everywhere, in particular, on the given interval and so the Mean Value Theorem can be used.

\[ g'(x) = 5 - \frac{2x}{2-x^2} \quad g(0) = -2.30685 \quad g(1) = 2 \]

Applying the Mean Value Theorem and solving gives,

\[ 5 - \frac{2c}{2-c^2} = g'(c) = \frac{g(1) - g(0)}{1-(0)} = 4.30685 \]

\[ -2c = -0.69315(2-c^2) \]

\[ 0.69315c^2 + 2c - 1.3863 = 0 \quad \Rightarrow \quad c = -3.46293, \ 0.57755 \]

This required the quadratic formula to solve with some “messy” coefficients but nothing you aren’t capable of doing! The first solution is not in the interval (and in fact the function doesn’t even exist at this point!) while the second solution is in the interval and the only value that satisfies the Mean Value Theorem is 0.57755.

5. This is not as difficult as it might seem at first glance. First let’s acknowledge that we must have \( f(a) = f(b) = 0 \) since \( f(x) \) has roots at \( x = a \) and at \( x = b \). Now simply apply the Mean Value Theorem to \( f(x) \) on \([a, b]\). Doing this gives,

\[ f'(c) = \frac{f(b) - f(a)}{b-a} = \frac{0}{b-a} = 0 \quad \Rightarrow \quad f'(c) = 0 \]

So, the derivative will be zero for some value \( a < c < b \) and so \( f(x) \) must have a critical point in the interval \([a, b]\).

6. A quick sketch is to the right. If we consider the lowest side to be the “front” portion we get the following set of equations.

Minimize : \( C = 15x + 18(2y + x) = 36y + 33x \)
Constraint : \( 100 = xy \)

Solving the constraint for \( y \) gives : \( y = \frac{250}{x} \). Plugging this into the cost function gives,

\[ C(x) = 36\left(\frac{100}{x}\right) + 33x = \frac{3600}{x} + 33x \quad C'(x) = -\frac{3600}{x^2} + 33 = \frac{33x^2 - 3600}{x^2} \quad C^*(x) = \frac{7200}{x^2} \]

From the first derivative we have the following critical points : \( x = \pm \sqrt{\frac{3600}{33}} = \pm 10.4447 \). Note that we won’t get zero as a critical point because the function doesn’t exist there and because having a zero length doesn’t make much sense! Also, because we are dealing with lengths of a rectangle the negative
critical point makes no physical sense and so the only critical point that actually makes sense is: 
\[ x = 10.4447 \].

From the second derivative (in particular that it will be positive for positive \( x \)) we can see that the cost function will always be concave up and so \( x = 10.4447 \) must be an absolute minimum. The dimensions are then: \[ x = 10.4447, \quad y = \frac{100}{10.4447} = 9.5742 \]

7. A quick sketch is to the right. In this case we want to minimize the square of the distance between (0,-2) and \((x,y)\), a point on the graph and the equation of the graph is the constraint.

\[ d^2 = x^2 + (y + 2)^2 \]

Now, solve the graph equation for \( x^2 \) and plug this into the square of the distance to get the function we'll differentiate.

\[ x^2 = 4 - \frac{y^2}{9} \]

\[ f(y) = 4 - \frac{y^2}{9} + y^2 + 4y + 4 = \frac{8}{9}y^2 + 4y + 8 \]

\[ f'(y) = \frac{16}{9}y + 4 \quad f''(y) = \frac{16}{9} \]

From the derivative we get a single critical point of: \( y = -\frac{9}{4} \) and from the second derivative (always positive) we can see that the function is always concave up and so this must give an absolute minimum distance. The points are then, \[ \left( \frac{\sqrt{85}}{4}, -\frac{9}{4} \right) \quad \& \quad \left( -\frac{\sqrt{85}}{4}, -\frac{9}{4} \right) \]

10. \[ \lim_{t \to -2} \frac{9 + 4t - e^{4t+8}}{t^3 + 4t^2 + 4t} = \lim_{t \to -2} \frac{4 - 4e^{4t+8}}{3t^2 + 8t + 4} = \lim_{t \to -2} \frac{-16e^{4t+8}}{6t + 8} = -16 = 4 \]

12. \[ \lim_{x \to -\infty} \left[ 3x \ln \left( 1 - \frac{2}{x} \right) \right] = \lim_{x \to -\infty} \frac{\ln \left( 1 - \frac{2}{x} \right)}{1/3x} = \lim_{x \to -\infty} \frac{-\frac{2}{x}}{\frac{1}{3x}} = \lim_{x \to -\infty} -\frac{6}{x} = -6 \]

13. \[ du = \left[ \cos(3z) - 3z \sin(3z) \right] dz \]

14. \[ du = -2xe^{-x^2} \sec \left( 1 + e^{-x^2} \right) \tan \left( 1 + e^{-x^2} \right) dx \]