#3. (2 pts) \[ \lim_{t \to 3} \frac{2t^2 + 5t - 3}{9 - t^2} = \lim_{t \to 3} \frac{(2t-1)(t+3)}{(3+t)(3-t)} = \lim_{t \to 3} \frac{2t-1}{3-t} = \frac{-7}{6} \]

#5. (3 pts)
\[
\begin{align*}
\lim_{y \to 1} \frac{2 - \sqrt{y^2 + 3}}{y^2 + 5y - 6} &= \lim_{y \to 1} \frac{2 - \sqrt{y^2 + 3}}{2 + \sqrt{y^2 + 3}} = \lim_{y \to 1} \frac{4 - (y^2 + 3)}{(y^2 + 5y - 6)(2 + \sqrt{y^2 + 3})} \\
&= \lim_{y \to 1} \frac{1 - y^2}{(y^2 + 5y - 6)(2 + \sqrt{y^2 + 3})} = \lim_{y \to 1} \frac{(1-y)(1+y)}{(y+6)(y-1)(2 + \sqrt{y^2 + 3})} \\
&= \lim_{y \to 1} \frac{-1+y}{(y+6)(2 + \sqrt{y^2 + 3})} = \frac{-2}{28} = -\frac{1}{14}
\end{align*}
\]

#6. For (a) we can just use the lower formula since \(11 \geq 6\) contains points on both sides in this interval. For (b) we’ll need to look at the two one-sided limits since \(x = 6\) is the cutoff point and no interval contains points on both sides of it.

(a) – NOT GRADED!!! \[\lim_{x \to 11} g(x) = \lim_{x \to 11} (x^2 - 3) = 118\]

(b) (3 pts) First the one sided limits.
\[
\begin{align*}
\lim_{x \to 6^{-}} g(x) &= \lim_{x \to 6^{-}} e^{2+x} = e^8 \quad \text{because } x < 6 \text{ in this case,} \\
\lim_{x \to 6^{+}} g(x) &= \lim_{x \to 6^{+}} (x^2 - 3) = 33 \quad \text{because } x > 6 \text{ in this case}
\end{align*}
\]

So, \(\lim_{x \to 6} g(x)\) doesn’t exist since the two one-sided limits are not the same.

#8. (2 pts) In both of these limits the numerator is staying fixed at 4 and as \(x\) approaches 7 (from either side) we can see that \(7-x\) is approaching zero. So, we have a fixed number divided by something increasingly smaller and so it should make some sense that both of these are going to either \(\infty\) or \(-\infty\) and this will depend upon the sign of the denominator

In the first case \(7-x\) is positive since \(x < 7\) and raising this to the fifth power will keep it positive. So, in this case we have a fixed positive number in the numerator divided by something increasingly smaller positive number and so the limit in this case will be \(\infty\).

In the second case \(7-x\) is negative since \(x > 7\) and raising this to the fifth power will keep it negative. So, in this case we have a fixed n number in the numerator divided by an increasingly smaller negative number and so the limit in this case will be \(-\infty\).

\[
\begin{align*}
\lim_{x \to 7^{-}} \frac{4}{(7-x)^5} &= \infty \\
\lim_{x \to 7^{+}} \frac{4}{(7-x)^5} &= -\infty
\end{align*}
\]
#1.
(a) \( f(-3) = -2 \) \( \lim_{x \to -3} f(x) = 2 \) \( \lim_{x \to -3} f(x) = 2 \) \( \lim_{x \to -3} f(x) = 2 \)
(b) \( f(0) = 0 \) \( \lim_{x \to 0^-} f(x) = 0 \) \( \lim_{x \to 0^-} f(x) = 0 \) \( \lim_{x \to 0^+} f(x) = 0 \)
(c) \( f(2) = \text{d.n.e} \) \( \lim_{x \to 2^-} f(x) = -4 \) \( \lim_{x \to 2^-} f(x) = -4 \) \( \lim_{x \to 2^+} f(x) = -4 \)
(d) \( \lim_{x \to 3} f(x) = 2 \) \( \lim_{x \to -3} f(x) = 2 \) \( \lim_{x \to -3} f(x) = 4 \)
\( \lim_{x \to -3} f(x) = \text{ doesn't exist b/c } \lim_{x \to -3} f(x) \neq \lim_{x \to -3} f(x) \)

#2.
(a) \( \lim_{x \to -2} [9g(x) - 2h(x)] = 9 \lim_{x \to -2} g(x) - 2 \lim_{x \to -2} h(x) = 9(-1) - 2(7) = -23 \)
(b) \( \lim_{x \to -2} [1 - 5f(x)g(x)] = \lim_{x \to -2} 1 - 5 \left( \lim_{x \to -2} f(x) \right) \left( \lim_{x \to -2} g(x) \right) = 1 - 5(8)(-1) = 41 \)
(c) \( \lim_{x \to -2} \frac{[f(x)]^3}{10 + h(x)} = \frac{\left( \lim_{x \to -2} f(x) \right)^3}{\lim_{x \to -2} 10 + \lim_{x \to -2} h(x)} = \frac{8^3}{10 + 7} = \frac{512}{17} \)

#4.
\( \lim_{z \to -1} \frac{(z - 4)(z + 2) + 2 - 3z}{z^2 + 5z + 4} = \lim_{z \to -1} \frac{z^2 - 5z - 6}{z^2 + 5z + 4} = \lim_{z \to -1} \frac{(z + 1)(z - 6)}{(z + 4)(z + 1)}(z + 4) = \lim_{z \to -1} \frac{z - 6}{z + 4} = \frac{-7}{3} \)

#7. Note that we can’t just cancel the h’s since once is inside the absolute value bars. So, we’ll use the hint and recall that,
\[
|h| = \begin{cases} h & \text{if } h \geq 0 \\ -h & \text{if } h < 0 \end{cases}
\]
With this we can do the two one sided limits to eliminate the absolute value bars.
\[
\lim_{h \to 0^-} \frac{|h|}{h} = \lim_{h \to 0^-} \frac{-h}{h} = \lim_{h \to 0^-} -1 = -1 \quad \text{because } h < 0 \text{ in this case}
\]
\[
\lim_{h \to 0^+} \frac{|h|}{h} = \lim_{h \to 0^+} \frac{h}{h} = \lim_{h \to 0^+} 1 = 1 \quad \text{because } h > 0 \text{ in this case}
\]
The two one-sided limits are not the same and so \( \lim_{h \to 0} \frac{|h|}{h} \) does not exist.
In both of these limits the numerator is staying fixed at -5 and as $x$ approaches -3 (from either side) we can see that $x+3$ is approaching zero. So, we have a fixed number divided by something increasingly smaller and so it should make some sense that both of these are going to either $\infty$ or $-\infty$. Also note that in this case because we’re squaring the denominator it will always be positive and so both of these limits will have a fixed negative numerator divided by an increasingly small positive number. Therefore, both will be $-\infty$.

Here are the official answers to this problem.

$$\lim_{t \to -3^-} \frac{-5}{(2t+6)^2} = -\infty \quad \quad \lim_{t \to -3^+} \frac{-5}{(2t+6)^2} = -\infty$$