Limits At Infinity
Evaluate each of the following limits.

1. \( \lim_{x \to \infty} \frac{(x - 7)(x - 2)}{5x^2 + x - 10} \)

2. \( \lim_{t \to -\infty} \frac{7t^4 + 6t^3 + 8t}{5 + 8t^2 - 4t^3} \)

3. \( \lim_{z \to \infty} \frac{z^5 - 7z^3 + 5}{2z^6 + 4z^2} \)

4. Evaluate \( \lim_{y \to \infty} \frac{\sqrt{6 + 2y^2}}{9y - 8} \) and \( \lim_{y \to -\infty} \frac{\sqrt{6 + 2y^2}}{9y - 8} \).

Continuity

5. Determine where the following function is NOT continuous.

\[ g(x) = \frac{x^2 + 7x + 1}{1 + \csc(2x)} \]

6. Use the Intermediate Value Theorem to show that somewhere in the interval \([-3, 0]\) there is a root of \( h(x) = 3x^2 + 2x - e^{-x} \). Note that you aren’t being asked to actually find the root, only show that one exists.

7. The population (in hundreds) of fish in a lake is given by,

\[ P(t) = 2 \sin \left( \frac{t}{2} \right) \cos(t) + 4 \]

where \( t \) is in months. Sometime in the first 5 months the population of fish will fall below 250. Use the Intermediate Value theorem to find a time span of no more than \( \frac{1}{2} \) of a month where the population does fall below 250. Note that there are multiple answers to this question and any of them will be accepted.

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Definition of the Derivative
For problems 8 – 11 use the definition of the derivative to compute the derivative of the given function.

8. \( g(x) = 4 - 15x \)

9. \( V(t) = \frac{2-t}{4+3t} \)

10. \( f(x) = 4x^2 - 2x + 1 \)

11. \( W(z) = \sqrt{1-4z} \)

Interpretation of the Derivative
For problems 12 – 14 use the derivatives found in the previous part to answer each question.

12. Is \( f(x) = 4x^2 - 2x + 1 \) increasing, decreasing or not changing at \( x = -2 \) ?

13. Find the equation of the tangent line to \( h(x) = 4 - 15x \) at \( x = 9 \)

14. Does \( V(t) = \frac{2-t}{4+3t} \) ever stop changing? If so when does it stop?

15. Below is the graph of the derivative of some function. Determine if the function is increasing, decreasing or not changing at the points: \( x = -2 \), \( x = 1 \) and \( x = 4 \)?