#1. (2 pts)  

\[ f''(y) = 4(12 - 2y)^3(-2)(y^2 - 3)^2 + (12 - 2y)^4(2)(y^2 - 3)(2y) \]

\[ = 4(12 - 2y)^3(y^2 - 3)[-2(y^2 - 3) + y(12 - 2y)] = 4(12 - 2y)^3(y^2 - 3)[-4y^2 + 12y + 6] \]

Now, to get the critical points we'll need to solve the following three equations,

\[ 12 - 2y = 0 \quad \Rightarrow \quad y = 6 \]
\[ y^2 - 3 = 0 \quad \Rightarrow \quad y = \pm \sqrt{3} = \pm 1.7321 \]
\[ 4y^2 - 12y - 6 = 0 \quad \Rightarrow \quad y = \frac{12 \pm \sqrt{288}}{8} = -0.4365, 3.4365 \]

So, we have a total of 5 critical points for this function.

#2. (2 pts)  

\[ G'(z) = 6z(7 - z^2)^{\frac{1}{2}} + (3z^2 + 1)(\frac{1}{2})(7 - z^2)^{-\frac{1}{2}}(-2z) = 6z(7 - z^2)^{\frac{1}{2}} - \frac{2z(3z^2 + 1)}{3(7 - z^2)^{\frac{3}{2}}} \]

\[ = \frac{18z(7 - z^2) - 2z(3z^2 + 1)}{3(7 - z^2)^{\frac{3}{2}}} = \frac{4z(31 - 6z^2)}{3(7 - z^2)^{\frac{3}{2}}} \]

From the denominator we see that \( z = \pm \sqrt{7} \) must be a critical point since the derivative won't exist at those points (but the function does!). Also, the derivative will be zero at,

\[ z = 0, z = \pm \sqrt{\frac{31}{6}} = \pm 2.2730 \]

This function has five critical points.

#8. (2 pts) We found the critical points to this function in #3 and those that are in [-4, -1] are : -2.2749. So, all we need to do is evaluate the function at this point and the end points.

\[ Q(-4) = -376 \quad Q(-2.2749) = 158.9892 \quad Q(-1) = 65 \]

The absolute maximum is 158.9892 at \( t = -2.2749 \) and the absolute minimum is -376 at \( t = -4 \).

#10. (2 pts) We’ll first need the critical points of this function that lie in [0,15].

\[ P'(t) = 8 - 50 \frac{2t}{t^2 + 6} = \frac{8t^2 - 100t + 48}{t^2 + 6} = \frac{4(2t - 1)(t - 12)}{t^2 + 6} \]

So, there are two critical points for this function, \( t = \frac{1}{2} \) and \( t = 12 \) and both lie in the interval so all we need to do now is some quick function evaluations.

\[ P(0) = 160.4120 \quad P(\frac{1}{2}) = 162.3709 \quad P(12) = 95.4682 \quad P(15) = 97.8791 \]

The maximum population is then 16,237.09 (fractions don’t make much sense, but I’ll keep them) at \( t = \frac{1}{2} \) and the minimum population is 9,546.82 at \( t = 12 \).
#13. (2 pts) \( f''(t) = 15t^4 - 20t^3 - 360t^2 = 5x^2 \left( 3x^2 - 4x - 72 \right) \) So, it looks like the critical points are,

\[ x = 0, \frac{2+\sqrt{65}}{3} = -4.2775, 5.6108 \]

I will leave it to you to actually draw a number line for this problem. Here are the derivative values that I used for mine.

\[ f'(-5) = 2875 \quad f'(-1) = -325 \quad f'(1) = -365 \quad f'(6) = 2160 \]

From this we get the following increasing/decreasing information and classification of critical points.

Increasing : \((-\infty,-4.2775), (5.6108,\infty)\) Decreasing : \((-4.2775,0), (0,5.6108)\)

\[ x = -4.2775 : \text{Rel. Max.} \quad x = 0 : \text{Neither} \quad x = 5.6108 : \text{Rel. Min.} \]

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Not Graded

#3. \( Q'(t) = -12t^3 + 36t^2 + 144t = -12t(t^2 - 3t - 12) \) \( \Rightarrow t = 0, \frac{3\sqrt{15}}{2} = 0, -2.2749, 5.2749 \)

#4. \( A'(t) = 5 - 21\cos(3t) \Rightarrow \cos(3t) = \frac{5}{21} \Rightarrow 3t = \cos^{-1} \left( \frac{5}{21} \right) = 1.3304 \)

\[ 3t = 1.3304 + 2\pi n \quad \& \quad 3t = 4.9528 + 2\pi n \quad n = 0, \pm 1, \pm 2, \ldots \]

\[ t = 0.4435 + \frac{2\pi n}{3} \quad \& \quad t = 1.6509 + \frac{2\pi n}{3} \quad n = 0, \pm 1, \pm 2, \ldots \]

#5. \( h'(x) = 2xe^{x^2-5} - 24xe^{-4x^2} = 2x \left( e^{x^2-5} - 12e^{-4x^2} \right) \)

Now, clearly \[ x = 0 \] is a critical point. Any others will come from solving,

\[ e^{x^2-5} - 12e^{-4x^2} = 0 \Rightarrow e^{x^2-5} = 12e^{-4x^2} \Rightarrow e^{5x^2-5} = 12 \]

\[ \Rightarrow 5x^2 - 5 = \ln(12) \Rightarrow x = \pm \sqrt{\frac{\ln(12)}{5}} \]

So, there are three critical points for this function.

#6. Rel. Max : \( b, d, f \) \quad Rel. Min. : \( c, e \) \quad Abs. Max. : \( d \) \quad Abs. Min. : \( g \)

#7. Not much to this problem. We found all the critical points in #1 and those that are in \([1, 4]\) are : \( y = \sqrt{3} \) and \( y = 3.4365 \). So, all we need to do is evaluate the function at these points and the end points.

\[ f(\sqrt{3}) = 0 \quad f(3.4365) = 53,623.9707 \]

\[ f(1) = 40,000 \quad f(4) = 43,264 \]
The absolute maximum is then 53,623.9707 at \( y = 3.4365 \) and the absolute minimum is 0 at \( y = -2 \) and \( y = \sqrt{3} \).

**#9.** All we are really asking here is what is the absolute maximum of this function in \([0, 3]\) and we know how to do that. In #4 we found the critical points of this function to be,

\[
t = 0.4435 + \frac{2\pi n}{3} \quad & \quad t = 1.6509 + \frac{2\pi n}{3} \quad n = 0, \pm 1, \pm 2, \ldots
\]

and the ones that are in \([0, 3]\) (the interval we’re interested in here) are,

\[
0.4435, 1.6509, 2.5379
\]

The function evaluations for this problem are,

\[
A(0) = 6 \quad A(0.4435) = 1.4186 \quad A(1.6509) = 21.0533
\]

\[
A(2.5379) = 11.8906 \quad A(3) = 18.1152
\]

So, the absolute maximum is 21.0533, and this corresponds, to $21,053.3 and so yes, the manager does get a bonus.

**#11.** In this problem we’re asking for the absolute minimum of the function in \([0, 5]\). From #3 we know that we’ll have two critical points in this region \((t = 0, 4)\) and we need to evaluate the function at these points and the endpoints. Here are all the relevant evaluations for this problem.

\[
Q(0) = 8 \quad Q(4) = 648 \quad Q(5) = 433
\]

So, the absolute maximum of 648 and so it does rise above 500 grams and so the process will stop during the first 5 hours.

**#12.** In #5 we found the critical points of this function to be \( t = -1.2235, 0, 1.2235 \). I will leave it to you to actually draw a number line for this problem. Here are the derivative values that I used.

\[
h'( -2 ) = -1.4715 \quad h'( -1 ) = 0.4029 \quad h'( 1 ) = -0.4029 \quad h'( 2 ) = 1.4715
\]

From this we get the following increasing/decreasing information and classification of critical points.

Increasing : \((-1.2235, 0), (1.2235, \infty)\)   Decreasing : \((-\infty, -1.2235), (0, 1.2235)\)

\[
x = -1.2235 : \text{Rel. Min.} \quad x = 0 : \text{Rel. Max.} \quad x = 1.2235 : \text{Rel. Min.}
\]