Divergence Test

If $\lim_{n\to\infty} a_n \neq 0$ then $\sum a_n$ will diverge

Geometric Series

The geometric series $\sum_{n=1}^{\infty} a\,r^{n-1} = \sum_{n=0}^{\infty} a\,r^n$ will converge if $|r| < 1$ and if it converges it's value will be $\frac{a}{1-r}$.

Integral Test

Suppose that $f(x)$ is a positive, decreasing function on the interval $[k,\infty)$ and that $f(n) = a_n$ then,

1. If $\int_k^{\infty} f(x)\,dx$ is convergent so is $\sum_{n=k}^{\infty} a_n$.
2. If $\int_k^{\infty} f(x)\,dx$ is divergent so is $\sum_{n=k}^{\infty} a_n$.

***p*-series Test**

The series $\sum_{n=1}^{\infty} \frac{1}{n^p}$ will converge if $p > 1$ and will diverge if $p \leq 1$.

Comparison Test

Suppose that we have two series $\sum a_n$ and $\sum b_n$ with $a_n, b_n \geq 0$ for all *n* and $a_n \leq b_n$ for all *n*. Then,

1. If $\sum b_n$ is convergent then so is $\sum a_n$.
2. If $\sum a_n$ is divergent then so is $\sum b_n$.

Limit Comparison Test

Suppose that we have two series $\sum a_n$ and $\sum b_n$ with $a_n, b_n \geq 0$ for all *n*. Define,

$$c = \lim_{n\to\infty} \frac{a_n}{b_n}$$

If *c* is positive (*i.e.* $c > 0$) and is finite (*i.e.* $c < \infty$) then either both series converge or both series diverge.

Alternating Series Test

Suppose that we have a series $\sum a_n$ and either $a_n = (-1)^n b_n$ or $a_n = (-1)^{n+1} b_n$ where $b_n \geq 0$ for all *n*. Then if,

1. $\lim_{n\to\infty} b_n = 0$ and,
2. $\{b_n\}$ is eventually a decreasing sequence

the series $\sum a_n$ is convergent

Ratio Test

Suppose we have the series $\sum a_n$. Define,

$$L = \lim_{n\to\infty} \left| \frac{a_{n+1}}{a_n} \right|$$

Then,

1. if $L < 1$ the series is absolutely convergent (and hence convergent).
2. if $L > 1$ the series is divergent.
3. if $L = 1$ the series may be divergent, conditionally convergent, or absolutely convergent.

Root Test

Suppose that we have the series $\sum a_n$. Define,

$$L = \lim_{n\to\infty} \sqrt[n]{|a_n|} = \lim_{n\to\infty} |a_n|^{\frac{1}{n}}$$

Then,

1. if $L < 1$ the series is absolutely convergent (and hence convergent).
2. if $L > 1$ the series is divergent.
3. if $L = 1$ the series may be divergent, conditionally convergent, or absolutely convergent.