#2. (2 pts) First use a double angle formula to rewrite the integral and then split it up.

\[ \int z \sin^2(3z) \, dz = \frac{1}{2} \int z (1 - \cos(6z)) \, dz = \frac{1}{2} \left[ \int z \, dz - \int z \cos(6z) \, dz \right] \]

We’ll use integration by parts on the second integral.

\[ u = z \quad du = dz \quad dv = \cos(6z) \, dz \quad v = \frac{1}{6} \sin(6z) \]

The integral is then,

\[ \int z \sin^2(3z) \, dz = \frac{1}{2} \int z \, dz - \frac{1}{2} \left[ \frac{1}{6} z \sin(6z) - \frac{1}{6} \int \sin(6z) \, dz \right] = \frac{1}{4} z^2 - \frac{1}{12} z \sin(6z) - \frac{1}{72} \cos(6z) + c \]

#3. (2 pts) This integral becomes a lot easier with the substitution \( u = \tan(x) \).

\[ \int \frac{\tan(x) \sec^2(x)}{\tan^2(x) + 12 \tan(x) + 32} \, dx = \int \frac{u}{u^2 + 12u + 32} \, du = \int \frac{u}{(u+4)(u+8)} \, du \]

So, we’ll need to do partial fractions on this integral.

\[ \frac{u}{(u+4)(u+8)} = \frac{A}{u+4} + \frac{B}{u+8} \quad \Rightarrow \quad u = A(u+8) + B(u+4) \]

\[ \begin{align*} u = -4: \quad -4 &= 4A \\
\quad A &= -1 \\

u = -8: \quad -8 &= -4B \quad \Rightarrow \quad B = 2 \end{align*} \]

\[ \int \frac{\tan(x) \sec^2(x)}{\tan^2(x) + 12 \tan(x) + 32} \, dx = \int \frac{2}{u+8} - \frac{1}{u+4} \, du = 2 \ln(8 + \tan(x)) - \ln(4 + \tan(x)) + c \]

#6. (2 pts) Not much to do here other than deal with the limits properly.

\[ \int_{1}^{\infty} x^3 e^{1-x^4} \, dx = \lim_{t \to \infty} \int_{1}^{t} x^3 e^{1-x^4} \, dx = \lim_{t \to \infty} \left( -\frac{1}{4} e^{1-t^4} \right) = \lim_{t \to \infty} \left( -\frac{1}{4} e^{1-t^4} + \frac{1}{4} \right) = \frac{1}{4} \]

So the integral converges and has a value of \( 1/4 \).

#8. (2 pts) This integral is a little different from those done in class. It has a division by zero issue as well as an infinite limit. We’ll need to split this up (at any point) and do each of the individual integrals as each one will be like one of those that we did in class.

\[ \int_{0}^{\infty} \frac{e^{1-\ln(x)}}{x} \, dx = \int_{0}^{1} \frac{e^{1-\ln(x)}}{x} \, dx + \int_{1}^{\infty} \frac{e^{1-\ln(x)}}{x} \, dx \]

\[ = \lim_{t \to 0^+} \int_{t}^{1} \frac{e^{1-\ln(x)}}{x} \, dx + \lim_{s \to \infty} \int_{1}^{s} \frac{e^{1-\ln(x)}}{x} \, dx \quad u = 1 - \ln(x) \]

\[ = \lim_{t \to 0^+} \left( -e^{1-\ln(x)} \right) + \lim_{s \to \infty} \left( e^{1-\ln(x)} - e \right) \]

Now,
\[
\lim_{t \to -\infty} e^{t \ln(t)} = e^{\lim_{t \to -\infty} t \ln(t)} = e^{1 - \infty} = \infty
\]
\[
\lim_{s \to \infty} e^{-s \ln(s)} = e^{\lim_{s \to \infty} -s \ln(s)} = e^{1 - \infty} = 0
\]

The first integral diverges and the second converges and so the whole integral will **diverge**.

**#9. (2 pts)** Because of the exponential in the denominator and the fact that the cosine term in the numerator won’t be that large we can guess that this will converge and so we’ll need to find a larger function that we can prove converges.

\[
\frac{4 + \cos^2(3 + x)}{e^x} \leq \frac{4 + 1}{e^x} \quad \text{b/c } 0 \leq \cos^2(3 + x) \leq 1
\]

\[
= 5e^{-x}
\]

I’ll leave it to you to verify that \(\int_{1}^{\infty} 5e^{-x} \, dx\) converges and so by the Comparison Test the original integral must also **converge**.

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**Not Graded**

**#1.** To do this integral we first need to rewrite things a little bit...

\[
\int \sin^3\left(\frac{\pi}{4}\right) \sec^4\left(\frac{\pi}{4}\right) \, dt = \int \frac{\sin^3\left(\frac{\pi}{4}\right)}{\cos^4\left(\frac{\pi}{4}\right)} \, dt = \int \frac{1 - \cos^2\left(\frac{\pi}{4}\right)}{\cos^4\left(\frac{\pi}{4}\right)} \sin\left(\frac{\pi}{4}\right) \, dt
\]

\[
= 4 \int \frac{1 - u^2}{u^4} \, du = 4\left(-\frac{1}{3}u^{-3} + u^{-1}\right) + c = 4\left(-\frac{1}{3}\sec^3\left(\frac{\pi}{4}\right) + \sec\left(\frac{\pi}{4}\right)\right) + c
\]

**#4.** This looks like the following trig substitution.

\[
y = \frac{3}{4}\sin \theta \quad dy = \frac{3}{4}\cos \theta \quad \sqrt{9 - 16y^2} = \sqrt{9 - 9\sin^2 \theta} = 3|\cos \theta| = 3\cos \theta
\]

The integral is then,

\[
\int (1 - y)\sqrt{9 - 16y^2} \, dy = \int (1 - \frac{3}{4}\sin \theta)(3\cos \theta)\left(\frac{3}{4}\cos \theta\right) d\theta = \frac{9}{4}\int (1 - \frac{3}{4}\sin \theta)\cos^2 \theta \, d\theta
\]

\[
= \frac{9}{4}\int 2\cos \theta \, d\theta - \frac{9}{16}\int \sin \theta \cos^2 \theta \, d\theta = \frac{9}{8}\int \cos(2\theta) \, d\theta - \frac{9}{16}\int \sin \theta \cos^2 \theta \, d\theta
\]

\[
= \frac{9}{8}\left(\theta + \frac{1}{4}\sin(2\theta)\right) + \frac{9}{16}\cos^3 \theta + c
\]

Now, from a right triangle we have \(\sin \theta = \frac{4\sqrt{3}}{5}, \quad \theta = \sin^{-1}\left(\frac{4\sqrt{3}}{5}\right), \quad \cos \theta = \frac{\sqrt{9 - 16y^2}}{3}, \quad \text{and so the integral is,}
\]

\[
\int (1 - y)\sqrt{9 - 16y^2} \, dy = \frac{9}{8}\left(\sin^{-1}\left(\frac{4\sqrt{3}}{5}\right) + \frac{4\sqrt{3}}{9}\sqrt{9 - 16y^2}\right) + \frac{\sqrt{9 - 16y^2}^3}{48} + c
\]
#5. \( u = x^5 \quad dv = x^4 \sin(1-x^3) \, dx \quad \Rightarrow \quad du = 5x^4 \quad v = \frac{1}{3} \cos(1-x^5) \)

\[ \int x^5 \sin(1-x^3) \, dx = \frac{1}{3} x^5 \cos(1-x^5) - \int x^4 \cos(1-x^3) \, dx = \frac{1}{3} x^5 \cos(1-x^5) + \frac{1}{3} \sin(1-x^5) + c \]

#7. There is a division by zero problem at \( x = 2 \) so we’ll need to split this up as follows.

\[ \int_0^5 \frac{x}{\sqrt{x^2 - 4}} \, dx = \lim_{t \to 2^+} \int_0^t \frac{x}{\sqrt{x^2 - 4}} \, dx + \lim_{s \to 2^+} \int_s^5 \frac{x}{\sqrt{x^2 - 4}} \, dx = \lim_{t \to 2^+} \frac{1}{4} \left( x^2 - 4 \right)^{\frac{3}{2}} \bigg|_0^t + \lim_{s \to 2^+} \frac{1}{4} \left( x^2 - 4 \right)^{\frac{3}{2}} \bigg|_s^5 \]

\[ = \lim_{t \to 2^+} \frac{3}{4} \left( t^2 - 4 \right)^{\frac{3}{2}} - \frac{3}{4} \left( -4 \right)^{\frac{3}{2}} + \lim_{s \to 2^+} \frac{3}{4} \left( 21 \right)^{\frac{3}{2}} - \frac{3}{4} \left( s^2 - 4 \right)^{\frac{3}{2}} = \frac{3}{4} \left( 21^{\frac{3}{2}} - (-4)^{\frac{3}{2}} \right) = 3.8189 \]

So, the integral converges and has the value shown above.

#10. It looks like this integral should behave like \( \frac{5}{x^2} = \frac{1}{x^2} \) and we know that \( \int_2^\infty \frac{1}{x^2} \, dx \) will diverge because \( p = \frac{1}{2} \leq 1 \) so we can guess that this will diverge and we’ll need to find a smaller function that we can prove diverges.

\[ \frac{\sqrt{x} + 5 \cos^4(3x)}{x - \sin^2 x} \geq \frac{\sqrt{x} + 0}{x - \sin^2 x} \quad \text{b/c} \quad 0 \leq 5 \cos^4(3x) \leq 5 \]

\[ \geq \frac{\sqrt{x}}{x - 0} \quad \text{b/c} \quad 0 \leq \sin^2 x \leq 1 \]

\[ = \frac{1}{\sqrt{x}} \]

So, by the Comparison Test we know that the original integral must diverge.

#11. For this problem we have \( f(x) = e^{1 + \cos(x)} \) and \( \Delta x = \frac{1}{2} \).

MidPoint Rule

\[ \int_{-1}^1 e^{1 + \cos(x)} \, dx \approx \frac{1}{2} \left[ f(-0.75) + f(0.25) + f(0.25) + f(0.75) \right] = 12.81307 \]

Trapezoid Rule

\[ \int_{-1}^1 e^{1 + \cos(x)} \, dx \approx \frac{1}{2} \left[ f(-1) + 2f(-0.5) + 2f(0) + 2f(0.5) + f(1) \right] = 12.56210 \]

Simpson’s Rule

\[ \int_{-1}^1 e^{1 + \cos(x)} \, dx \approx \frac{1}{6} \left[ f(-1) + 4f(-0.5) + 2f(0) + 4f(0.5) + f(1) \right] = 12.73526 \]

For comparison’s sake,

\[ \int_{-1}^1 e^{1 + \cos(x)} \, dx = 12.73012 \]