Arc Length
For problems 1 & 2 find the length of the given curve.

1. \( y = (x + 2)^{\frac{3}{2}} - 1, \ 0 \leq x \leq 6 \)

2. \( x = y^2 + 1, \ 1 \leq x \leq 10 \). Assume that \( y \geq 0 \) for this problem.

3. Set up, but do not evaluate an integral that will give the length of the following curve. Yes, I realize that there are no limits given, you will need to determine them. I also realize that this function is not given in the same form as the previous two, dealing with that is part of the problem.

\[
\frac{x^2}{25} + y^2 = 1
\]

Surface Area
For problems 4 & 5 find the surface area of the region obtained by rotating the function about the given axis.

4. \( x = e^y, \ 0 \leq y \leq 1 \) about the \( y \)-axis.

5. \( y = 1 + 3x^2, -1 \leq x \leq 2 \) about the \( y \)-axis.

6. Set up, but do not evaluate, the integral that will give the surface area obtained by rotating \( y = \cos^2 x, \ 0 \leq x \leq \pi \) about,

(a) the \( x \)-axis.
(b) the \( y \)-axis.

Parametric Equations and Curves
For problems 7 & 8 do each of the following.

(a) Eliminate the parameter for the parametric equations and sketch the parametric curve clearly indicating the direction in which the curve is traced out as \( t \) increases.

(b) Determine what (if any) limits exist on the values of \( x \) and \( y \).

(c) If the curve is traced out more than once give a range of \( t \)'s for which the curve is traced out exactly once. Tracing out a curve exactly once means that no portion of the curve will be retraced (in either direction) in the range of \( t \)'s given.

7. \( x = t + 1, \ y = 1 - t^2 \)

8. \( x = e^t, \ y = \frac{1}{2}e^t \)

Continued on Back ⇒
For problems 9 & 10 suppose that a particle is tracing out a path given by the following parametric equations. Completely describe the motion of the particle as \( t \) varies in the given interval. This means that you need to do (a) – (c) from the previous problems above as well as,

(d) If any portion of the curve is retraced determine how many times the path is traced out.

9. \( x = 4 - 2 \cos^2 \left(6t\right), \quad y = 3 \sin^2 \left(6t\right), \quad -2\pi \leq t \leq 7\pi \)

10. \( x = \cos \left(\frac{t}{3}\right), \quad y = 1 + 2 \cos^4 \left(\frac{t}{3}\right), \quad -2\pi \leq t \leq 7\pi \)