3. (3 pts) \( \frac{dy}{dt} = 3t^2 - 9 \) \( \frac{dx}{dt} = 2t - 1 \) \( \frac{dy}{dx} = \frac{3t^2 - 9}{2t - 1} \)

The value(s) of \( t \) that give the point are,
\[
2 = t^2 - t - 4 \quad \Rightarrow \quad t^2 - t - 6 = (t - 3)(t + 2) = 0 \quad \Rightarrow \quad t = -2, 3
\]
\[
1 = t^3 - 9t + 1 \quad \Rightarrow \quad t^3 - 9t = t(t - 3)(t + 3) = 0 \quad \Rightarrow \quad t = -3, 0, 3
\]

So, we’ll have one tangent line at \( t = 3 \). Here’s the work for this.

\[
m = \left. \frac{dy}{dx} \right|_{t=3} = \frac{18}{5}
\]
\[
y = 1 + \frac{18}{5}(x - 2) = \frac{18}{5}x - \frac{31}{5}
\]

5. (2 pts) \( A = \int 2\pi y \, ds = \int_0^4 2\pi (1-t^8) \sqrt{(2t e^{6t} + 6t^2 e^{6t})^2 + 25t^8} \, dt \)

7. (NOT GRADED) \( r = 3 - 6 \sin \theta \)

8. (2 pts) \( r = 9 \cos \theta \) - Circle of radius 4.5 centered at (4.5,0).

10. (3 pts) A sketch is to the right. The intersection points are,
\[
5 \cos \theta = 4 - 3 \cos \theta \quad \Rightarrow \quad \cos \theta = \frac{1}{2} \quad \Rightarrow \quad \theta = -\frac{\pi}{3}, \frac{\pi}{3}
\]

Note that we needed to use a negative value for the first to get the region to properly trace out. The area is then,
\[
A = \frac{1}{2} \int_{-\frac{\pi}{3}}^{\frac{\pi}{3}} (5 \cos \theta)^2 - (4 - 3 \cos^2 \theta) \, d\theta
\]
\[
= 4 \int_{-\frac{\pi}{3}}^{\frac{\pi}{3}} 2 \cos^2 \theta + 3 \cos \theta - 2 \, d\theta
\]
\[
= 4 \int_{-\frac{\pi}{3}}^{\frac{\pi}{3}} \cos (2\theta) + 3 \cos \theta - 1 \, d\theta = 14\sqrt{3} - \frac{8}{3} \pi = 15.8711
\]

Not Graded
Math 2414  Homework Set 5  10 Points

1. \( \frac{dy}{dt} = 2t + 3 \quad \frac{dx}{dt} = 6t - 9 \)
\[ \frac{dy}{dx} = \frac{2t + 3}{6t - 9} \quad \frac{d^2y}{dx^2} = \frac{2(6t - 9) - 6(2t + 3)}{(6t - 9)^2} = -\frac{36}{(6t - 9)^3} \]

2. \[ \frac{dy}{dt} = 6 \quad \frac{dx}{dt} = -2\sin(2t) - 3 \quad \frac{dy}{dx} = -\frac{6}{2\sin(2t) + 3} \]
\[ \left. \frac{dy}{dx} \right|_{t=0} = -2 \quad (x, y)_{t=0} = (1, -5) \quad y = -5 - 2(x - 1) = -2x - 3 \]

4. Note that the distance traveled and the length are probably going to be two different quantities. The
length is the distance traveled on one trace while the distance traveled is the distance that the particle
travels over the whole range of \( t \)'s. To do this problem we will essentially need to redo the work for the
last couple of problems in the previous homework set so we can get the limits to compute the length of
the curve. Once we have that and the number of traces we'll know the distance traveled since it will be
nothing more than the length times the number of traces. So, here is the work.

\[ \cos^2(2t) = \frac{y^2}{\pi} \quad \sin^2(2t) = x + 1 \]
\[ 1 = \cos^2(2t) + \sin^2(2t) = \frac{y^2}{\pi} + x + 1 \quad \Rightarrow \quad x = -\frac{y^2}{\pi} \]
\[ -1 \leq x \leq 0 \quad -3 \leq y \leq 3 \]
\[ x = 3 : \cos(2t) = 1 \quad 2t = 0 + 2\pi n \quad t = \pi n \]
\[ x = -3 : \cos(2t) = -1 \quad 2t = \pi + 2\pi n \quad t = \frac{\pi}{2} + \pi n \]

So, we have a portion of a parabola that is traced out once in the range \( 0 \leq t \leq \frac{\pi}{2} \) and will trace out a
total of 10 times. Now the length is,

\[ L = \int_{0}^{\frac{\pi}{2}} \sqrt{\left(4\sin(2t)\cos(2t)\right)^2 + \left(-6\sin(2t)\right)^2} \, dt = \int_{0}^{\frac{\pi}{2}} \sqrt{16\sin^2(2t)\cos^2(2t) + 36\sin^2(2t)} \, dt \]
\[ = \int_{0}^{\frac{\pi}{2}} 2\sin(2t) \sqrt{4\cos^2(2t) + 9} \, dt \]

Now, we are working on \( 0 \leq t \leq \frac{\pi}{2} \) and in this range we have \( 0 \leq 2t \leq \pi \) and so we know that
\( \sin(2t) \geq 0 \) in the range of \( t \)'s that we're working on here so we can drop the absolute value bars and
then the integral can be done with the trig substitution \( \cos(2t) = \frac{1}{2} \tan \theta \). Doing the trig substitution gives,

\[ L = -\frac{9}{2} \int_{\tan^{-1}\left(-\frac{3}{2}\right)}^{\tan^{-1}\left(-\frac{1}{2}\right)} \sec^3 \theta \, d\theta = -\frac{9}{4} \left( \sec \theta \tan \theta + \ln |\sec \theta + \tan \theta| \right)_{-\frac{3}{2}}^{0.588} = 6.4187 \]

The distance traveled is then: \( D = 10(6.4187) = 64.187 \).
6. 
\[ r \left( 2r^3 \sin \theta \cos \theta \right) = r \left( 3 - \cos \theta \right) \]
\[ 2r^2 \left( r \sin \theta \right) \left( r \cos \theta \right) = 3r - r \cos \theta \quad \Rightarrow \quad 2xy \left( x^2 + y^2 \right) = 3\sqrt{x^2 + y^2} - x \]

9. The curve will go through the origin at,
\[ 0 = 3 - 6 \sin \theta \quad \Rightarrow \quad \sin \theta = \frac{1}{2} \quad \Rightarrow \quad \theta = \frac{\pi}{6}, \frac{5\pi}{6} \]
The area of the inner loop is then,
\[ A = \int_{\frac{\pi}{6}}^{\frac{5\pi}{6}} \frac{1}{2} \left( 3 - 6 \sin \theta \right)^2 \, d\theta = \frac{1}{2} \int_{\frac{\pi}{6}}^{\frac{5\pi}{6}} 9 - 36 \sin^2 \theta \, d\theta = 9\pi - \frac{27\sqrt{3}}{2} = 4.8916 \]

11. For this one we need the answer to 10 as the area is simply the portion of the circle that we didn’t use in 10. So, the area here is,
\[ A = \left( \text{Area of circle} \right) \left( \text{Answer} \right) = \pi \left( \frac{5}{2} \right)^2 - 15.8711 = 3.7639 \]