Math 2415  Homework Set 1 - Solutions  10 Points

1. (2 pts) Level Curve \(( z = c ) : c = 5x - y^2 \rightarrow x = \frac{1}{5}(c + y^2)\) So the level curve is a **parabola**.

Trace \(( x = a ) : z = 5a - y^2\) So this trace is a **parabola** in the \(yz\)–plane.

Trace \(( y = b ) : z = 5x - b^2\) So this trace is a **line** in the \(xz\)–plane.

5. (2 pts) \(\vec{r}(\theta) = \langle 12 \cos \theta, 20, 12 \sin \theta \rangle\)

6. (2 pts) \(\vec{r}(t) = (1-t)(-7, 4, 2) + t(-3, -5, 1), 0 \leq t \leq 1\). Note that the range of \(t\)'s is important.

Without the range you get the line that goes through the two points and not the line segment starting at \(P\) and ending at \(Q\).

8. (2 pts) For the first term we’ll need to use L’Hospitals Rule. The rest can be done directly.

\[
\lim_{t \to 2} \vec{r}(t) = \lim_{t \to 2} \frac{\sin(t-2)}{t^2-4} \vec{i} + \lim_{t \to 2} e^{6-3t} \vec{j} - \lim_{t \to 2} (9t^2 + 8t - 4) \vec{k} \\
= \lim_{t \to 2} \frac{\cos(t-2)}{2t} \vec{i} + \lim_{t \to 2} e^{6-3t} \vec{j} - \lim_{t \to 2} (9t^2 + 8t - 4) \vec{k} = \frac{1}{4} \vec{i} + \vec{j} - 48 \vec{k}
\]

10. (2 pts) Don’t forget all the basic integration rules you learned in Calc I and Calc II. You will be asked to do those on occasion in this class. The first uses integration by parts and the third uses a simple trig identity. I’ll leave the details to you to verify for each of these.

\[
\int t \cos(t) dt = t \sin(t) - \int \sin(t) dt = t \sin(t) + \cos(t) \\
\int e^{8t} dt = \frac{1}{8} e^{8t} \\
\int \sin^2(4t) dt = \frac{1}{2} \left(1 - \cos(8t)\right) dt = \frac{1}{2} \left(t - \frac{1}{8} \sin(8t)\right) \\
\int \vec{r}(t) dt = \langle t \sin(t) + \cos(t), \frac{1}{8} e^{8t}, \frac{1}{2} \left(t - \frac{1}{8} \sin(8t)\right) \rangle + \vec{c}
\]


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2. Level Curve \(( z = c ) : 2x^2 - 4y^2 + 2z^2 - 4 = 0\) So the level curve is a **hyperbola**.

Trace \(( x = a ) : -4y^2 + 2z^2 + 2a^2 - 4 = 0\) So this trace is a **hyperbola** in the \(yz\)–plane.

Trace \(( y = b ) : 2x^2 + 2z^2 - 4b^2 - 4 = 0\) So this trace is a **circle** in the \(xz\)–plane.

3. \(\vec{r}(x) = \langle x, e^{2x} - \sin(x+1) \rangle\)

4. \(\vec{r}(x,z) = \langle x, \cos(x+z) - z^2 + x^4, z \rangle\)
7. (a) \( \vec{r}_u \cdot \vec{r}_v = (2v)(0) + (-u^3)(uv) + (-1)(-8) = 8 - vu^4 \)

(b) \[ \vec{r}_u \times \vec{r}_v = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 2v & -u^3 & -1 \\ 0 & uv & -8 \end{vmatrix} = 2v \vec{i} - u^3 \vec{j} - 8 \vec{k} \]

\[ = 8u^3 \vec{i} + 2uv^2 \vec{k} - (16v \vec{j}) - (uv \vec{i}) = (8u^3 + uv) \vec{i} + 16v \vec{j} + 2uv^2 \vec{k} \]

(c) \[ \| \vec{r}_u \times \vec{r}_v \| = \sqrt{(8u^3 + uv)^2 + 256v^2 + 4u^2v^4} \]

9. \( \vec{r}'(t) = \left( \frac{1-t^2}{(t^2+1)^2}, \frac{6}{6t+1}, \cos(3t) - 3t \sin(3t) \right) \)

11. First we need,

\[ \vec{r}'(t) = 2t e^{-t} \vec{i} - 3 \vec{j} + \left[ \sin(\pi t) + \pi t \cos(\pi t) \right] \vec{k} \]

\[ \vec{r}(1) = \vec{i} - 2 \vec{j} \quad \vec{r}'(1) = 2 \vec{i} - 3 \vec{j} - \pi \vec{k} \]

The tangent line is then,

\[ \vec{r}(t) = (1, -2, 0) + t(2, -3, -\pi) = (1 + 2t - 2t - \pi t) \]

12. \[ \vec{r}'(t) = \left( \frac{3}{t}, \sqrt{6}, t \right) \quad \| \vec{r}'(t) \| = \sqrt{\frac{9+6t^2+t^4}{t^2}} = \sqrt{\frac{(t^2+3)^2}{t^2}} = \sqrt{t^2+3} \]

You did recall that \( \sqrt{a^2} = |a| \) right? Also, we can drop the absolute value bars because of the assumption that \( t > 0 \). The unit tangent is then,

\[ T(t) = \frac{\vec{r}'(t)}{\| \vec{r}'(t) \|} = \left( \frac{t}{t^2+3}, \frac{3}{t^2+3}, \frac{\sqrt{6}t}{t^2+3} \right) \]

For the unit normal we have,

\[ T'(t) = \left( \frac{-6t}{(t^2+3)^2}, \frac{\sqrt{6}(3-t^2)}{(t^2+3)^2}, \frac{6t}{(t^2+3)^2} \right) \]

\[ \| T'(t) \| = \sqrt{\frac{36t^2}{(t^2+3)^4} + \frac{6(3-t^2)^2}{(t^2+3)^4} + \frac{36t^2}{(t^2+3)^4}} = \sqrt{\frac{6t^4+36t^2+54}{(t^2+3)^4}} = \sqrt{\frac{6(t^2+3)^2}{(t^2+3)^4}} = \frac{\sqrt{6}}{t^2+3} \]

The unit normal is then,
\[ \vec{N}^\prime(t) = \frac{T^\prime(t)}{\|T^\prime(t)\|} = \frac{t^2 + 3}{\sqrt{6}} \begin{pmatrix} -6t & \sqrt{6} \left(3 - t^2\right) \\ (t^2 + 3)^2 & (t^2 + 3)^2 \end{pmatrix} = \begin{pmatrix} -\sqrt{6}t & (3 - t^2) \\ (t^2 + 3)^2 & t^2 + 3 \end{pmatrix} \]