4. (2 pts) This function is not continuous at (0,0) so let’s see if we can find a couple of paths that give different values for the limit.

\[
\begin{align*}
y \text{-axis (} x = 0 \text{): } & \lim_{(x,y) \to (0,0)} \frac{x^4y^2}{2x^2 + 6y^3} = \lim_{(x,y) \to (0,0)} \frac{0}{6y^3} = 0 \\
y = x^4: & \lim_{(x,y) \to (0,0)} \frac{x^4y^2}{2x^2 + 6y^3} = \lim_{(x,y) \to (0,0)} \frac{x^4(x^8)}{2x^2 + 6(x^{12})} = \lim_{(x,y) \to (0,0)} \frac{x^{12}}{8x^{12}} = \lim_{(x,y) \to (0,0)} \frac{1}{8} = \frac{1}{8}
\end{align*}
\]

So, we have two paths that give different values of the limit and so we know that \( \lim_{(x,y) \to (0,0)} \frac{x^4y^2}{2x^2 + 6y^3} \) doesn’t exist.

6. (2 pts)

\[
\begin{align*}
\frac{\partial w}{\partial x} &= y \cos(xy) e^{2y^2z^2} \\
\frac{\partial w}{\partial y} &= x \cos(xy) e^{2y^2z^2} + 2 \sin(xy) e^{2y^2z^2} \\
\frac{\partial w}{\partial z} &= 2z \sin(xy) e^{2y^2z^2}
\end{align*}
\]

7. \( f_u = 2u \ln\left(s^2 - 8t^4\right) + 4 \sec^2(4u) \quad f_v = 0 \quad f_s = \frac{2su^2}{s^2 - 8t^4} \quad f_t = -\frac{32r^3u^2}{s^2 - 8t^4} \)

9. (2 pts)

\[
\begin{align*}
z_x &= \frac{1}{x} + 6y^2x^5 \\
z_y &= -\frac{1}{y} + 2yx^6 - 4 \\
z_{xx} &= -\frac{1}{x^2} + 30y^2x^4 \\
z_{xy} &= 12yx^5 \\
z_{yx} &= 12yx^5 \\
z_{yy} &= \frac{1}{y^2} + 2x^6
\end{align*}
\]

11. (2 pts) The key here is to recall that they can be done in any order we to make life simpler do the s derivative followed by 4 t derivatives.

\[
\begin{align*}
f_s &= -\frac{t^6}{\sqrt{1-2s}} \\
f_{tt} &= -\frac{6t^5}{\sqrt{1-2s}} \\
f_{ttt} &= -\frac{30t^4}{\sqrt{1-2s}} \\
f_{tttt} &= -\frac{120t^3}{\sqrt{1-2s}} \\
f_{ttttt} &= f_{ttttt} = -\frac{360t^2}{\sqrt{1-2s}}
\end{align*}
\]

Not Graded

1. \( L = \int_1^{10} \|r'(t)\|\,dt = \int_1^{10} t^2 + 3 \,dt = \left(\frac{1}{3} t^3 + 3 \ln(t)\right)^{10}_1 = \frac{99}{3} + 3 \ln(10) \)
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2. Not much to do here. This function is continuous at (0,0) so:
\[ \lim_{(x,y) \to (0,0)} \frac{\cos(x) - \sin(y)}{xy + x^2 - y^2 + 4} = \frac{1}{4} \]

3. This function is not continuous at (0,0) so let’s see if we can find a couple of paths that give different values for the limit. I’ll leave it to you to verify that both the x-axis and y-axis give the same value and so we can only use one of them. I’ll use the x-axis.

\[ x = 0 \] : 
\[ \lim_{(x,y) \to (0,0)} \frac{(7x - 4y)^3}{2y^3 + x^3} = \lim_{(x,0) \to (0,0)} \frac{(7x)^3}{x^3} = \lim_{(x,0) \to (0,0)} 343 = 343 \]

\[ y = x \] : 
\[ \lim_{(x,y) \to (0,0)} \frac{(7x - 4y)^3}{2y^3 + x^3} = \lim_{(x,x) \to (0,0)} \frac{(3x)^3}{3x^3} = \lim_{(x,x) \to (0,0)} 9 = 9 \]

So, we have two paths that give different values of the limit and so we know that 
\[ \lim_{(x,y) \to (0,0)} \frac{7x - 4y}{2y^3 + x^3} \]

doesn’t exist.

5. \( g_x = -\frac{4y^3z^2}{x^5} + \frac{1}{\sqrt{4z + 2x}} \)
\( g_y = \frac{3y^2z^2}{x^4} - 21 \cos^2(7y) \sin(7y) \)
\( g_z = \frac{2y^3z}{x^4} + \frac{2}{\sqrt{4z + 2x}} \)

8.
\[ 4y^2z^3 \frac{\partial z}{\partial x} - \sec^2(1-x) = 18z^5 \frac{\partial z}{\partial x} \implies \frac{\partial z}{\partial x} = \frac{\sec^2(1-x)}{4y^2z^3 - 18z^5} \]
\[ 2yz^4 + 4y^2z^3 \frac{\partial z}{\partial y} = 18z^5 \frac{\partial z}{\partial y} \implies \frac{\partial z}{\partial y} = \frac{2yz^4}{18z^5 - 4y^2z^3} \]

10.
\[ h_s = -4s^3t^3 \sin(t^3s^4) + \frac{4s^3}{t^2} \]
\[ h_t = -3t^2s^4 \sin(t^3s^4) - \frac{2s^4}{t^3} \]
\[ h_{ss} = -12s^2t^3 \sin(t^3s^4) - 16s^6t^6 \cos(t^3s^4) + \frac{12s^2}{t^2} \]
\[ h_{st} = -12s^3t^2 \sin(t^3s^4) - 12s^7t^5 \cos(t^3s^4) - \frac{8s^3}{t^3} \]
\[ h_{ss} = -12t^2s^3 \sin(t^3s^4) - 12t^5s^7 \cos(t^3s^4) - \frac{8s^3}{t^3} \]
\[ h_{tt} = -6ts^4 \sin(t^3s^4) - 9t^4s^8 \sin(t^3s^4) + \frac{6s^4}{t^4} \]

12. In this case we don’t need to worry so much about order as we did with #11 as none of the terms will drop out so we’ll just go in the given order.
\[
\frac{\partial u}{\partial z} = -\frac{x^4y^2}{z^2} + 7x^3z^6y^2 \\
\frac{\partial^2 u}{\partial z^2} = \frac{2x^4y^3}{z^3} + 42x^3z^5y^2 \\
\frac{\partial^3 u}{\partial y\partial z^2} = \frac{3x^4y^2}{z^3} + 84x^3z^5y \\
\frac{\partial^4 u}{\partial x\partial y\partial z^2} = \frac{12x^3y^2}{z^3} + 252x^2z^5y \\
\frac{\partial^5 u}{\partial x^2\partial y\partial z^2} = \frac{36x^2y^2}{z^3} + 504xz^5y \\
\frac{\partial^6 u}{\partial y\partial x^2\partial y\partial z^2} = \frac{18x^2y^{\frac{1}{2}}}{z^3} + 504xz^5
\]