Triple Integrals with Spherical Coordinates
For problems 1 and 2 you must use spherical coordinates to do the problems.

1. $\iiint_E (8x + 4y) \, dV$ where $E$ is the region below the sphere of radius 3 and inside the cone $\varphi = \frac{\pi}{3}$.

2. $\iiint_E (x^2 + y^2 + z^2)^{\frac{3}{2}} \, dV$ where $E$ is the region that lies in the first octant and between the spheres of radius 1 and 4.

3. Evaluate the following integral by first converting to spherical coordinates.

$$\int_{-\sqrt{3}}^{0} \int_{0}^{\sqrt{3-x^2}} \int_{-\sqrt{3-x^2-z^2}}^{\sqrt{3-x^2-z^2}} x \, dz \, dy \, dx$$

Change of Variables
For problems 4 and 5 find the Jacobian of the transformation.

4. $x = uv - 5u^3 + 7v, \quad y = 3v - 8u$

5. $x = \mu \cos \alpha, \quad y = \mu \sin \alpha$

For problems 6 – 8 find and graph the image of the set $R$ under the given transformation. Note that the point of these examples is not necessarily to transform $R$ into a “nice” region. Instead all we’re trying to do is apply some transformations to regions.

6. $R$ is the triangle with vertices $(0, 0), (-2, 2), (6, 2)$ and the transformation is $x = 3v, \quad y = \sqrt{2-u}$.
   Note that for each side you’ll want the range of possible $x$ or $y$ values to in order to get a range of possible $u$ or $v$ values that are needed for the transformed sides. This will make the sketch a little easier maybe……

7. $R$ is the region bounded by $xy = 2, \quad xy = 6, \quad x = y, \quad y = 4$ and the transformation is $x = u, \quad y = \frac{2v}{u}$.

8. $R$ is the disk given by $x^2 + y^2 \leq 1$ and the transformation is $u = ax, \quad v = by$.

9. If $R$ is the parallelogram with vertices $(2, -1), (-2, 1), (4, 5), (0, 7)$ use the transformation $x = 2(u - 2v), \quad y = -(u + 12v)$ to convert the region and evaluate the integral $\iint_R 3x - y \, dA$.

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10. If $R$ is the ellipse $x^2 + \frac{y^2}{4} = 1$ determine a transformation that will convert this into a circle of radius 1 and use that transformation to evaluate $\int \int_R xy \, dA$.

Note that you’ve already seen how to turn a circle of radius 1 into an ellipse so use that as a guide to determine this transformation.

**Surface Area**

For problems 11 and 12 find the area of the given surface.

11. The portion of the plane $8x + 4y + 3z = 24$ that lies in the first octant.

12. The portion of $z = 4x^2 + 4y^2 - 1$ that lies below $z = 11$.

13. In class I gave you the formula for the surface area of $z = f(x, y)$ that lies above a region $D$ in the $xy$-plane. Use a modification of this formula to find the surface area of the portion of the surface $y = 6 + 7x + 2z^2$ that lies in front of the triangle in the $xz$-plane with vertices $(0,0)$, $(6,3)$ and $(0,3)$. 