**Line Integrals of Vector Fields**

For problems 1 and 2 evaluate \( \int_C \vec{F} \cdot d\vec{r} \) for the given vector field, \( \vec{F} \), and the given curve, \( C \).

1. \( \vec{F}(x, y) = xy \, \hat{i} - (1 - y) \, \hat{j} \) where \( C \) is the left half of \( \frac{x^2}{16} + \frac{y^2}{9} = 1 \) and with counter clockwise motion.

2. \( \vec{F}(x, y, z) = yz \, \hat{i} - (x + z^2) \, \hat{j} + \cos(4y) \, \hat{k} \) and \( C \) is given by \( \vec{r}(t) = (9 - t) \, \hat{i} + t^2 \, \hat{j} - 6t \, \hat{k}, \ 0 \leq t \leq 1 \)

**Fundamental Theorem for Line Integrals**

3. Evaluate \( \int_C \nabla f \cdot d\vec{r} \) for \( f(x, y, z) = x^2 \sin(5x^3 - y) + 2xyz \) and \( C \) is given by \( \vec{r}(t) = (t^2 + t, 2t^3 + 6, 7t^3), \ -1 \leq t \leq 3 \).

4. For the vector field in #1 above is it possible to determine if \( \int_C \vec{F} \cdot d\vec{r} \) is independent of path? If so, is the integral independent of path and if it is not possible to determine explain why not.

**Conservative Vector Fields**

For problems 5 and 6 determine if the vector field, \( \vec{F} \), is conservative or not. If it is conservative find the potential function for the vector field.

5. \( \vec{F} = (8xy^3 + 3x^2y) \, \hat{i} + (12x^2y^2 - x^3) \, \hat{j} \)

6. \( \vec{F} = (6x + 3x^2y^4) \, \hat{i} - (8y - 4x^3y^3 + 7) \, \hat{j} \)

For problems 7 – 9 find the potential function for the vector field and then evaluate \( \int_C \vec{F} \cdot d\vec{r} \) for the given curve \( C \).

7. \( \vec{F} = (6x + 3x^2y^4) \, \hat{i} - (8y - 4x^3y^3 + 7) \, \hat{j} \) and \( C \) is the line segment from \((1, -3)\) to \((4, 0)\).

8. \( \vec{F} = (-4x - e^{2y} \cos(4y - x)) \, \hat{i} + (1 + 2e^{2y} \sin(4y - x) + 4e^{2y} \cos(4y - x)) \, \hat{j} \) and \( C \) is the bottom half of the circle of radius 4 that is centered at \((-1, 2)\) starting at the right and ending at the left.

9. \( \vec{F} = 2x \ln(y^2z) \, \hat{i} - (27y^2z^4 - 2x^2) \, \hat{j} + \left( \frac{x^2}{y} - 36y^3z^3 \right) \, \hat{k} \) and \( C \) is \( \vec{r}(t) = (4t - 8) \, \hat{i} + 2t^2 \, \hat{j} + \hat{k}, \ 1 \leq t \leq 2 \).

Continued on Back ⇒
Green’s Theorem
For problems 10 and 11 evaluate the line integral (a) directly and (b) using Green’s Theorem. Assume all curves have the positive orientation.

10. \[ \oint_C (2xy + 3y) \, dx - (3x - x^2) \, dy \] where \( C \) is the circle of radius 3 centered at the origin with positive orientation.

11. \[ \oint_C x^2 \, dy - (4y - 8xy) \, dx \] where \( C \) is the triangle with vertices (0,0), (3,9) and (3,-1) with positive orientation.