Math 2415 Homework Set 1 10 Points

Functions of Several Variables
Identify the level curves and traces of each of the following
1. \( y^2 = 9z^2 - 7x^2 \)
2. \( x^2 + 9y^2 - z = 2 \)

Vector Functions
For problems 3 – 6 write down the vector function for the given curve or surface.
3. \( y = e^{7-x} + 2 \sin(8z) \)
4. \( f(x) = x \cos(x) \)
5. \( h(x, z) = \ln(x + z) - 4z \)
6. A circle of radius 6 centered on the \( y \)-axis at \( y = -4 \)

7. Find the vector equation for the line segment starting at \( P = \langle -1, 4, 2 \rangle \) and ending at \( Q = \langle 1, -8, -3 \rangle \).

8. Given \( \vec{r}_u = \langle 8u + v^3 \rangle \hat{i} - u^2 \hat{k} \) and \( \vec{r}_v = 4v \hat{i} - e^v \hat{j} + \ln(uv) \hat{k} \) compute,
   \( \text{(a)} \quad \vec{r}_u \cdot \vec{r}_v \quad \text{(b)} \quad \vec{r}_u \times \vec{r}_v \quad \text{(c)} \quad \| \vec{r}_u \times \vec{r}_v \| \)

Note: Yes, I realize we didn’t do any of this stuff in class, but the dot product, cross product and magnitude of vectors will be very important down the road so dig back into your Calc II notes are recall how to do these!

Calculus with Vector Functions
9. If \( \vec{r}(t) = \langle \cos(2t + 6), 3t - t^2, \frac{t^2 - 9}{12 + 4t} \rangle \) compute \( \lim_{t \to 3} \vec{r}(t) \).
10. Find the derivative of \( \vec{r}(t) = \langle 4t^3 e^{-2t}, 4t - t^2, \ln[\sec(t)] \rangle \)
11. Integrate \( \int 8t^2 \hat{i} - te^{6t} \hat{j} + \cos^2(2t) \hat{k} \; dt \)

Tangent Vectors and Normal Vectors
12. Find the tangent line to \( \vec{r}(t) = \langle \cos(\pi t), 5t, t^2 - 6t + 3 \rangle \) at \( t = 4 \).

13. Find the unit tangent and unit normal vectors for the vector function \( \vec{r}(t) = \langle \frac{1}{2}t^2, \sqrt{6}t, \ln(t^3) \rangle \).

Note: For reasons that will eventually be apparent you should simplify your answer as much as possible.