Surface Integrals
For problems 1 – 3 evaluate the surface integral.

1. \[ \iint_S yz + 4y^2 + 2xy \, dS \] where \( S \) is the portion of \( 4x + 8y + 2z = 6 \) that lies in the first octant. \( S \) is only the plane and does not include any of the sides.

2. \[ \iint_S \sqrt{1-x} \, dS \] where \( S \) is the portion of \( x = 1 - 3y^2 - 3z^2 \) that lies in above of \( x = -8 \). \( S \) is just the elliptic paraboloid and does NOT include the cap.

3. \[ \iint_S y + 1 \, dS \] where \( S \) is the portion of the cylinder \( x^2 + z^2 = 9 \) and bounded by \( y = 1 \) and \( y = 6 - x \). Note that \( S \) includes both the ends of the cylinder as well.

Surface Integrals of Vector Fields
For problems 4 and 5 evaluate \( \iiint_S \vec{F} \cdot d\vec{S} \) for the given vector field and surface.

4. \( \vec{F}(x, y, z) = 7x\vec{i} - (2y + 14)^2 \vec{j} + 7z\vec{k} \) and \( S \) is the part of the cone \( y = 2x^2 + 2z^2 - 7 \) that lies in behind of \( y = 1 \) and oriented in the direction of the negative \( y \)-axis. \( S \) is just the elliptic paraboloid and does NOT include the cap.

5. \( \vec{F}(x, y, z) = x\vec{i} + 3(6 - y)\vec{j} + 2z\vec{k} \) and \( S \) is the surface from problem 3. Use the positive orientation for this problem.

Stokes' Theorem
6. Use Stokes' Theorem to evaluate \( \iint_S \text{curl} \vec{F} \cdot d\vec{S} \) where \( \vec{F} = e^{x^2 - yz}\vec{i} + 4y\vec{j} - (3 - 8x)\vec{k} \)
and \( S \) is the part of the sphere \( x^2 + y^2 + z^2 = 10 \) with \( x \geq 0 \) and inside the cylinder \( y^2 + z^2 = 1 \), oriented in the direction of the positive \( x \)-axis.

7. Use Stokes' Theorem to evaluate \( \int_C \vec{F} \cdot d\vec{r} \) where \( \vec{F} = xz\vec{i} + 5x\vec{j} - 2y\vec{k} \) and \( C \) is the circle \( x^2 + y^2 = 4 \) at \( z = 1 \) and \( C \) is oriented counter clock-wise when viewed from above.

Hint: You’ll need an easy to work with surface whose intersection with the plane \( z = 6 \) is the circle \( x^2 + y^2 = 5 \) and by this point in the semester you’ve worked many times with one particular kind of function that will do this.

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Divergence Theorem

8. Use the Divergence Theorem to evaluate \( \iiint \vec{F} \cdot d\vec{r} \) where \( \vec{F} = x^3 \vec{i} + 3yz^2 \vec{j} - 3y^2z \vec{k} \) and \( S \) is the portion of the surface bounded by the two spheres \( x^2 + y^2 + z^2 = 4 \) and \( x^2 + y^2 + z^2 = 36 \) that is in the first octant.