

Separable Differential Equations

For problems 1 & 2 find the solution to the given IVP and determine the interval of validity for the solution. Any approximate answers must be to at least the 4th decimal place and you may need computational aids in finding some of the intervals of validity.

$$1. \quad y' = \frac{4t + t y^2}{y + y t^2} \quad y(0) = -2$$

$$2. \quad y' = e^{y-1}(6x+2) \quad y(0) = -4$$

3. Solve the following differential equation, find the interval of validity for the solution. Any approximate answers must be to at least the 4th decimal place.

$$y' = \frac{1-3x}{2+y} \quad y(-1) = 0$$

Modeling, Part I

For problems 4 & 5 you MUST set up and solve the appropriate IVP(s) in order to receive any credit for the problem. Any decimals must be to at least the 4th decimal place.

4. A 750 gallon tank contains 500 gallons of water with 25 ounces of pollution dissolved in it. Polluted water with a concentration of $c(t) = 20 + 8e^{\frac{3-t}{40}}$ ounces/gallon is flowing into the tank at a rate of 5 gallons/min and a well mixed solution flows out at a rate of 5 gallons/min. How much pollution is in the water after 2 hours of operation?

5. A 1200 liter tank initially contains 900 liters of water with 20 grams of salt dissolved in it. Salt water with a concentration of 4 grams/liter flows into the tank at a rate of 9 liters/hr and a well mixed solution flows out at a rate of 6 liters/hr. This will continue until right before the tanks starts to overflow. At that point in time the inflow of salt water is turned off and pure water is pumped into the system at a rate of 2 liters/hr and a well mixed solution flows out at a rate of 6 liters/hr.

At what time will the amount of salt in the tank reach 500 grams? Note that I want the time as measured from the very start of the process.

6. Take the same situation from #5 and once the amount of salt reaches 500 grams the pure water is turned off salt water with a new concentration of 7 grams/liter is pumped into the system with a flow rate of 10 liters/hr. The outflow is increased to 10 liters/hr. Set up, but do not solve, the IVP that governs this situation.