#1. (2 pts) This is a fairly simple differential equation to separate and solve.

\[
\int \frac{y}{4x^2} \, dy = \int \frac{1}{4t^2} \, dt \quad \rightarrow \quad \frac{1}{2} \ln (4 + y^2) = \frac{1}{2} \ln (1 + t^2) + c \quad \Rightarrow \quad \ln (4 + y^2) = \ln (1 + t^2) + \ln 8 = \ln (8(1 + t^2))
\]

Finally, to get an explicit solution all we need to do is exponentiate both sides.

\[
y(t) = \pm \sqrt{4 + 8t^2} \quad \Rightarrow \quad y(t) = -2\sqrt{1 + 2t^2}
\]

This solution exists for all \( t \) so the interval of validity is \(-\infty < t < \infty\).

#2. (3 pts) This is a fairly simple differential equation to separate and solve.

\[
\int e^{-y} \, dy = \int 6x + 2 \, dx \quad \rightarrow \quad -e^{-y} = 3x^2 + 2x + c \quad \Rightarrow \quad e^{-y} = e^5 - 2x - 3x^2
\]

It’s also pretty simple to find the explicit solution.

\[
y(x) = 1 - \ln \left( e^5 - 2x - 3x^2 \right)
\]

Now, for the interval of validity we need to worry about logarithms of zero and negative numbers so we’ll need to know the roots of

\[
e^5 - 2x - 3x^2 = 0 \quad \Rightarrow \quad x = -\frac{2 \pm \sqrt{4 + 12x^2}}{12x} = -7.3748, \ 6.7081
\]

The only interval in which the polynomial is positive is below and is also the actual interval of validity (it contains \( t = 0 \)).

\(-7.3748 < t < 6.7081\)

#5. (5 pts) Here’s the two IVP’s for the system.

\[
Q_1' = (9)(4) - \frac{6Q_1}{900 + 3t} = 36 - \frac{2Q_1}{300 + t} \quad Q_1(0) = 20
\]

\[
Q_2' = (2)(0) - \frac{6Q_2}{1200 - 4(t - t_c)} = -\frac{3Q_2}{600 - 2(t - t_c)} \quad Q_2(t_c) = Q_1(t_c)
\]

Note that for the second IVP we know that the tank will overflow after \( t_c = 100 \) hours of operation and we know that at this point the initial volume for this case will be 1200 liters. We won’t be able to get the initial condition for the second IVP until the first is solved. Here is the solution to the first IVP.

\[
Q_1(t) = 12\left(300 + t\right) - \frac{3580 \left(300^2\right)}{\left(300 + t\right)^2} \quad Q_1(100) = 2786.25
\]

The second IVP and its solution is then,

\[
Q_2' = -\frac{3Q_2}{800 - 2t} \quad Q_2(100) = 2786.25 \quad \Rightarrow \quad Q_2(t) = 0.18958(800 - 2t)^\frac{3}{2}
\]

Now, we are after the time in which this reaches 500 grams. So solving gives,

\[
500 = 0.18958(800 - 2t)^\frac{3}{2} \quad \Rightarrow \quad t = 304.5547
\]
#3. Again, this is a simple differential equation to separate and solve.

\[
\int 2 + y \, dy = \int 1 - 3x \, dx \quad \rightarrow \quad \frac{1}{2}y^2 + 2y = x - \frac{3}{2}x^2 + c \quad \Rightarrow \quad y^2 + 4y - (3x^2 + 2x + 5) = 0
\]

Using the quadratic formula to find the explicit solution (the "+" gives the i.c...) gives,

\[
y(x) = \frac{4 \pm \sqrt{16 + 4(-3x^2 + 2x + 5)}}{2} \quad \Rightarrow \quad y(x) = 2 + \sqrt{-3x^2 + 2x + 9}
\]

Now for the interval of validity. Using the quadratic formula on the quadratic under the radical we can see that we will only have positive numbers under the radical is the range below which is also the interval of validity for this problem.

\[
-1.4305 = -\frac{1 - 2\sqrt{7}}{3} \leq x \leq \frac{1 + 2\sqrt{7}}{3} = 2.0972
\]

#4. Here’s the IVP and the solution (I'll leave the solution details to you to check).

\[
Q'(t) = 5 \left(20 + 8e^{\frac{3-t}{40}}\right) - \frac{5Q}{500} = 100 + 40e^{\frac{3-t}{40}} - \frac{Q}{100} \quad Q(0) = 25
\]

\[
Q(t) = 10,000 - \frac{8000}{3}e^{\frac{3-t}{40}} + \left(\frac{8000}{3}e^{\frac{3-t}{40}} - 9975\right)e^{-\frac{t}{100}}
\]

After 2 hours (don’t forget to convert to minutes....) of operation there is \(Q(120) = 7718.2218\) ounces.

#6. Here’s the IVP

\[
Q'_3 = (10)(7) - \frac{10Q_3}{381.7812} = 70 - 0.02619Q_3 \quad Q_3(304.5547) = 500
\]

The volume will now be fixed since the inflow and outflow is the same and will be the volume when the second situation from #5 is over, i.e. \(1200 - 4(304.5547 - 100) = 381.7812\) hrs.