#2. (2 pts)

\[ W = \left| \begin{array}{cc} 2x^2 & g \\ 4x & g' \end{array} \right| = 2x^2 g' - 4x g = x^4 e^{-3x} \quad \rightarrow \quad g' - \frac{2}{x} g = \frac{1}{2} x^2 e^{-3x} \]

This is a simple linear differential equation so I’ll let you verify my solution.

\[ g(x) = -\frac{1}{6} x^2 e^{-3x} + cx^2 \]

#6. (4 pts) I’ll let you verify that the complimentary solution is: \( y_c(t) = c_1 e^{7t} + c_2 \).

The guess for the particular solution and its derivatives are,

\[ Y_p = (At + B) \cos(2t) + (Dt + E) \sin(2t) \]
\[ Y_p' = (A + 2E + 2Dt) \cos(2t) + (D - 2B - 2At) \sin(2t) \]
\[ Y_p'' = (4D - 4At - 4B) \cos(2t) + (-4A - 4E - 4Dt) \sin(2t) \]

Plugging this into the differential equation and simplifying gives,

\[ (-4A + 14D) t \cos(2t) + (-14A - 4D) t \sin(2t) + (7A - 4B + 4D + 14E) \cos(2t) \]
\[ + (-4A - 14B + 7D - 4E) \sin(2t) = (106t + 1) \cos(2t) \]

Setting coefficients equal gives,

\[
\begin{align*}
    t \cos(2t) &: -4A + 14D = 106 & A &= -2 \\
    t \sin(2t) &: -14A - 4D = 0 & B &= \frac{425}{106} \\
    \cos(2t) &: 7A - 4B + 4D + 14E = 1 & D &= 7 \\
    \sin(2t) &: -4A - 14B + 7D - 4E = 0 & E &= \frac{23}{106} \\
\end{align*}
\]

The general solution is then,

\[ y(t) = c_1 e^{7t} + c_2 + (-2t + \frac{425}{106}) \cos(2t) + (7t + \frac{23}{106}) \sin(2t) \]

#7. (4 pts) I’ll let you verify that the complimentary solution is: \( y_c(t) = c_1 e^{-3t} + c_2 e^{4t} \).

The guess for the particular solution and its derivatives are,

\[ Y_p = (At + B) e^{-3t} \quad Y_p' = (-4At + A - 4B) e^{-3t} \quad Y_p'' = (16At - 8A + 16B) e^{-3t} \]

Plugging this into the differential equation and simplifying gives,

\[ 8Ate^{-3t} + (-9A + 8B) e^{-3t} = 8te^{-4t} \]

Setting coefficients equal gives,

\[
\begin{align*}
    te^{-4t} &: 8A = 8 & A &= 1 \\
    e^{-4t} &: -9A + 8B = 0 & B &= \frac{9}{8} \\
\end{align*}
\]

The general solution is then,

\[ y(t) = c_1 e^{-3t} + c_2 e^{4t} + (t + \frac{9}{8}) e^{-4t} \]

Plugging in the initial conditions gives,
\[ c_1 + c_2 + \frac{2}{3} = 0 \quad \Rightarrow \quad c_1 = -2 \]
\[ -3c_1 + 4c_2 - \frac{7}{2} = 6 \quad \Rightarrow \quad c_2 = \frac{7}{8} \]

The actual solution is then,
\[ y(t) = -2e^{-3t} + \frac{7}{8}e^{4t} + \left(t + \frac{9}{8}\right)e^{-4t} \]

---

**Not Graded**

**#1.** Compute the Wronskian.

\[
W(y_1, y_2) = \begin{vmatrix}
    e^{t_1} & e^{t_2} \\
    e^{r_1 t} & e^{r_2 t}
\end{vmatrix} = r_2 e^{(r_1 + r_2)t} - r_1 e^{(r_1 + r_2)t} = (r_2 - r_1)e^{(r_1 + r_2)t} \neq 0 \quad \text{b/c} \quad r_2 \neq r_1
\]

So they are a fundamental set of solutions and the general solution is what I claimed it to be and the general solution is \( y(t) = c_1e^{nt} + c_2e^{nt} \).

**#3.** I'll let you verify that the complimentary solution is : \( y_c(t) = c_1e^{2t} + c_2e^{3t} \).

The guess for the particular solution and its derivatives are,
\[
Y_p = Ae^{2t} \quad Y_p' = 7Ae^{2t} \quad Y_p'' = 49Ae^{2t}
\]

Plugging this into the differential equation and simplifying gives,
\[ 125Ae^{2t} = 10e^{2t} \quad \Rightarrow \quad 125A = 10 \quad \Rightarrow \quad A = \frac{2}{25} \]

The general solution is then,
\[ y(t) = c_1e^{2t} + c_2e^{3t} + \frac{2}{25}e^{2t} \]

**#4.** I'll let you verify that the complimentary solution is : \( y_c(t) = c_1e^{-t}\cos(6t) + c_2e^{-t}\sin(6t) \).

The guess for the particular solution and its derivatives are,
\[
Y_p = A\cos(3t) + B\sin(3t) \quad Y_p' = -3A\sin(3t) + 3B\cos(3t) \quad Y_p'' = -9A\cos(3t) - 9B\sin(3t)
\]

Plugging this into the differential equation and simplifying gives,
\[ (28A + 6B)\cos(3t) + (-6A + 28B)\sin(3t) = 20\cos(3t) \]

Setting coefficients equal gives,
\[
\cos(3t) : \quad 28A + 6B = 20 \quad \Rightarrow \quad A = \frac{28}{41} \]
\[ \sin(3t) : \quad -6A + 28B = 0 \quad \Rightarrow \quad B = \frac{6}{41} \]

The general solution is then,
\[ y(t) = c_1e^{-t}\cos(6t) + c_2e^{-t}\sin(6t) + \frac{28}{41}\cos(3t) + \frac{6}{41}\sin(3t) \]
I’ll let you verify that the complimentary solution is: \( y_c(t) = c_1 e^{8t} + c_2 te^{8t} \).

The guess for the particular solution and its derivatives are,

\[
\begin{align*}
Y_p &= At^2 + Bt + C & Y_p' &= 2At + B & Y_p'' &= 2A \\
Y_p &= At^2 + Bt + C & Y_p' &= 2At + B & Y_p'' &= 2A
\end{align*}
\]

Plugging this into the differential equation and simplifying gives,

\[
64At^2 + (-32A + 64B)t + 2A - 16B + 64C = 128t^2 + 16t - 4
\]

Setting coefficients equal gives,

\[
\begin{align*}
t^2 : & \quad 64A = 128 \quad \Rightarrow \quad A = 2 \\
t^1 : & \quad -32A + 64B = 16 \quad \Rightarrow \quad B = \frac{5}{4} \\
t^0 : & \quad 2A - 16B + 64C = -4 \quad \Rightarrow \quad C = \frac{3}{16}
\end{align*}
\]

The general solution is then,

\[
y(t) = c_1 e^{8t} + c_2 te^{8t} + 2t^2 + \frac{5}{4} t + \frac{3}{16}
\]