Math 3301 **Homework Set 9** **10 Points**

IVP's with Step Functions

Use Laplace transforms to solve the given IVP. In the partial fraction stage all quadratics that can be factored with integer coefficients must be factored.

1. $y'' - 6y' + 5y = 3u_{10}(t) - 11 \qquad y(0) = 4,\ y'(0) = -9$

2. $y'' + 4y' + 4y = \mathbf{e}^{-2t} - u_3(t)\mathbf{e}^{6-2t} \qquad y(0) = -2,\ y'(0) = -4$

3. $4y'' + y = -7u_4(t)\cos(3t - 12) \qquad y(0) = 0,\ y'(0) = 4$

Dirac-Delta Function

Use Laplace transforms to solve the given IVP. In the partial fraction stage all quadratics that can be factored with integer coefficients must be factored.

4. $4y'' - y' - 3y = 10\delta(t-2) \qquad y(0) = -4,\ y'(0) = -3$

5. $y'' - 10y' + 29y = -8\delta(t-2) - 4u_6(t) \qquad y(0) = 0,\ y'(0) = 0$

Convolution Integrals

6. Find the Laplace Transform of $f(t) = \int_0^t \tau^2 \mathbf{e}^{2\tau - 2t}\mathbf{e}^{8\tau}\, d\tau$.

7. Use a convolution integral (and yes, I expect you to evaluate the integral…) to find the inverse transform of

$$H(s) = \frac{7}{s^3 + 100s}$$

8. Find the solution to the following IVP in terms of $g(t)$.

$$y'' + 7y' + 12y = g(t) \qquad y(0) = 0,\ y'(0) = 3$$