1. (3 pts) Here’s the IVP’s we need for this problem. Note that I’m using a time frame of months here and so all per week quantities will need to be multiplied by 4 to get them into a per month quantity.

\[ P' = rP \quad P(0) = 300 \quad P(2) = 1200 \]

\[ P' = rP + 30(4) - 40(4) = rP - 40 \quad P(0) = 300 \]

Solving the first and applying the initial condition gives the following solution which we can then apply the second condition,

\[ P(t) = 300e^{rt} \quad 1200 = 300e^{2r} \quad r = \frac{1}{2} \ln(4) = \ln(4)^{\frac{1}{2}} = \ln(2) \]

The second IVP is now,

\[ P' = \ln(2)P - 40 \quad P(0) = 300 \]

I’ll leave it to you to verify that the solution is,

\[ P(t) = \frac{40}{\ln(2)} + 242.2922e^{\ln(2)t} \]

From this we can see that the insects will survive because everything is positive and the exponential will go to infinity as \( t \to \infty \).

3. (3 pts) Here’s the IVP for this case.

\[ v' = 9.8 - \frac{30}{20} v = 9.8 - \frac{3}{2} v \quad v(0) = 0.75 \]

I’ll leave it to you to verify the solution to this.

\[ v(t) = 6.5333 - 5.7833e^{-\frac{3}{2}t} \]

To determine when it hits the ground we can set this equal to 5 and solve for \( t \).

\[ 5 = 6.5333 - 5.7833e^{-\frac{3}{2}t} \quad \Rightarrow \quad 0.26513 = e^{-\frac{3}{2}t} \quad \Rightarrow \quad t = 0.8850 \]

The height function is,

\[ s(t) = \int 6.5333 - 5.7833e^{-\frac{3}{2}t} dt \quad s(0) = 0 \quad \Rightarrow \quad s(t) = 6.5333t + 3.8556e^{-\frac{3}{2}t} - 3.8556 \]

The bridge is then \( s(0.8850) = 2.9487 \text{ m} \) above the ground.

4. (2 pts) The equilibrium solutions are: \( y = -8 \), \( y = -4 \), and \( y = 0 \). From a sketch of the solutions we can see the following classifications.

\[ y = 0 : \text{ Semi-stable} \]
\[ y = -4 : \text{ Asymp. Stable} \]
\[ y = -8 : \text{ Unstable} \]
6. (2 pts) We just need to run through the formulas using \( f(t, y) = y + t^2 - \sin(y) \). Here’s the results for \( h = 0.4 \).

<table>
<thead>
<tr>
<th>( t )</th>
<th>2.4</th>
<th>2.8</th>
</tr>
</thead>
<tbody>
<tr>
<td>( f'_n )</td>
<td>10.3430134013</td>
<td>17.8871924346</td>
</tr>
<tr>
<td>Approx.</td>
<td>11.1372053605</td>
<td>18.2920823344</td>
</tr>
</tbody>
</table>

Here’s the results for \( h = 0.2 \).

<table>
<thead>
<tr>
<th>( t )</th>
<th>2.2</th>
<th>2.4</th>
<th>2.6</th>
<th>2.8</th>
</tr>
</thead>
<tbody>
<tr>
<td>( f'_n )</td>
<td>10.3430134013</td>
<td>13.5599105577</td>
<td>18.2479656363</td>
<td>21.9159513489</td>
</tr>
<tr>
<td>Approx.</td>
<td>9.0686026803</td>
<td>11.7805847918</td>
<td>15.4301779190</td>
<td>19.8133681888</td>
</tr>
</tbody>
</table>

For \( h = 0.4 \) we have \( y(2.8) \approx 18.2920823344 \) and for \( h = 0.2 \) we have \( y(2.8) \approx 19.8133681888 \).

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2. The new IVP that we’ll need for this new situation is,

\[
P'_2 = \ln(2) P_2 + 30(4) - 40(4) - 120(4) = \ln(2) P_2 - 520 \quad P_2(1.5) = P_1(1.5) = 743.0136
\]

I’ll let you verify that the solution to this IVP is,

\[
P(t) = \frac{520}{\ln(2)} - 2.5413e^{\ln(2)t}
\]

So, it looks like the insects will now die out (although just barely it seems as \( c \) is only just switched over to negative!). Setting equal to zero and solving gives that they will die at \( t = 8.2056 \). So, they will die out after about 33 weeks (or so...).

5. The equilibrium solutions are \( y = 0 \) and \( y = 2 \). From a sketch of the solutions we can see the following classifications.

- \( y = 2 \): Asymp. Stable
- \( y = 0 \): Unstable