Eigenvalues and Eigenvectors
Find the eigenvalues and eigenvectors of the given matrix.

1. \( A = \begin{bmatrix} \frac{1}{2} & 5 \\ \frac{1}{2} & 3 \end{bmatrix} \)

2. \( A = \begin{bmatrix} -2 & 1 \\ -1 & -4 \end{bmatrix} \)

3. \( A = \begin{bmatrix} 5 & -5 \\ 10 & -9 \end{bmatrix} \)

Systems of Differential Equations
Convert the given system into a system of differential equation and give your answer in matrix form.

4. \( 2y'' - 9y' - 6y = 0 \quad y(0) = 3, \quad y'(0) = 8 \)

5. \( y'' + 14y'' - y' = 0 \quad y(0) = 9, \quad y'(0) = -6, \quad y''(0) = 3 \)

Real, Distinct Eigenvalues
6. Find the general solution to the following system.
\[ \ddot{x}' = \begin{bmatrix} 2 & 3 \\ 4 & 9 \end{bmatrix} \ddot{x} \]

For problems 7 & 8 solve the system, sketch the phase portrait for the system and determine the stability of the equilibrium solution.

7. \( \ddot{x}' = \begin{bmatrix} -12 & -15 \\ 2 & 1 \end{bmatrix} \ddot{x} \quad \ddot{x}(0) = \begin{bmatrix} 0 \\ -4 \end{bmatrix} \)

8. \( \ddot{x}' = \begin{bmatrix} 4 & 1 \\ 7 & -2 \end{bmatrix} \ddot{x} \quad \ddot{x}(0) = \begin{bmatrix} -3 \\ -1 \end{bmatrix} \)

Complex Eigenvalues
For problems 9 solve the system, sketch the phase portrait for the system and determine the stability of the equilibrium solution.

9. \( \ddot{x}' = \begin{bmatrix} -2 & -5 \\ 4 & 2 \end{bmatrix} \ddot{x} \quad \ddot{x}(0) = \begin{bmatrix} 7 \\ -4 \end{bmatrix} \)

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10. Answer each of the following questions about the given IVP.
(a) Convert the IVP into a system of differential equations and write your answer in matrix form.
(b) Solve the system and use this solution to find the solution to the original 2nd order IVP.
(c) Sketch the phase portrait for the system and determine the stability of the equilibrium solution.

\[ 4y'' - 4y' + 17y = 0 \quad y(0) = 5, \quad y'(0) = -1 \]

Repeted Eigenvalues
For problem 11 & 12 solve the system, sketch the phase portrait for the system and determine the stability of the equilibrium solution.

11. \[ \ddot{x}' = \begin{bmatrix} 1 & -4 \\ 4 & 9 \end{bmatrix} \ddot{x} \quad \ddot{x}(0) = \begin{bmatrix} 0 \\ -2 \end{bmatrix} \]

12. \[ \ddot{x}' = \begin{bmatrix} -\frac{5}{4} & \frac{3}{2} \\ -\frac{3}{8} & \frac{1}{4} \end{bmatrix} \ddot{x} \quad \ddot{x}(0) = \begin{bmatrix} 4 \\ 1 \end{bmatrix} \]