2. (2 pts)

\[ W = \begin{vmatrix} x^3 & g \\ -3x^4 & g' \end{vmatrix} = x^3 g' + 3x^4 g = x^5 e^x \quad \rightarrow \quad g' + \frac{3}{x} g = x^2 e^x \]

This is a simple linear differential equation so I’ll let you verify my solution.

\[ g(x) = x^2 e^x - x^3 e^x + cx^{-3} \]

3. (2 pts) I’ll leave it to you to verify that \( y_c(t) = c_1 e^{-10t} + c_2 te^{-10t} \). The guess for the particular and its derivatives are,

\[ Y_p = At^3 + Bt^2 + Ct + D \quad Y'_p = 3At^2 + 2Bt + C \quad Y''_p = 6At + 2B \]

Plugging this into the differential equation and simplifying gives,

\[ 100At^3 + (60A + 100B)t^2 + (6A + 40B + 100C)t + 2B + 20C + 100D = 50t^3 - 4t \]

Setting coefficients equal and solving gives,

- \( t^3 : \quad 100A = 50 \quad A = \frac{1}{2} \)
- \( t^2 : \quad 60A + 100B = 0 \quad B = -\frac{1}{10} \)
- \( t^1 : \quad 6A + 40B + 100C = -4 \quad C = \frac{1}{250} \)
- \( t^0 : \quad 2B + 20C + 100D = 0 \quad D = -\frac{1}{250} \)

The general solution is then,

\[ y(t) = c_1 e^{-10t} + c_2 te^{-10t} + \frac{1}{2} t^3 - \frac{1}{10} t^2 + \frac{1}{250} t - \frac{1}{250} \]

6. (3 pts) I’ll leave it to you to verify that \( y_c(t) = c_1 e^t + c_2 e^{3t} \). The guess for the particular solution and its derivatives are,

\[ Y_p = e^{3t} \left[ A \cos(t) + B \sin(t) \right] \quad Y'_p = e^{3t} \left[ (3A + B) \cos(t) + (-A + 3B) \sin(t) \right] \]

\[ Y''_p = e^{3t} \left[ (8A + 6B) \cos(t) + (-6A + 8B) \sin(t) \right] \]

Plugging this into the differential equation and simplifying gives,

\[ e^{3t} \left[ (-11A - 3B) \cos(t) + (3A - 11B) \sin(t) \right] = 5e^{3t} \cos(t) \]

Setting coefficients equal and solving gives,

- \( e^{3t} \cos(t) : \quad -11A - 3B = 5 \quad A = -\frac{11}{26} \)
- \( e^{3t} \sin(t) : \quad 3A - 11B = 0 \quad B = 0 \)

The general solution is then,

\[ y(t) = c_1 e^t + c_2 e^{3t} + e^{3t} \left[ -\frac{11}{26} \cos(t) - \frac{3}{26} \sin(t) \right] \]

7. (3 pts) I’ll leave it to you to verify that \( y_c(t) = c_1 \cos(2t) + c_2 \sin(2t) \). The guess for the particular solution and its derivatives are,

\[ Y_p = \left( At^2 + Bt + C \right) e^{6t} \quad Y'_p = \left( 6At^2 + (2A + 6B)t + B + 6C \right) e^{6t} \]

\[ Y''_p = \left( 36At^2 + (24A + 36B)t + 2A + 12B + 36C \right) e^{6t} \]
Plugging this into the differential equation and simplifying gives,
\[(2A + 12B + 40C)e^{6t} + (24A + 40B)te^{6t} + 40A^2t^2e^{6t} = 200t^2e^{6t}\]
Setting coefficients equal and solving gives,
\[e^{6t} : \quad 2A + 12B + 40C = 0 \quad \Rightarrow \quad A = 5\]
\[te^{6t} : \quad 24A + 40B = 0 \quad \Rightarrow \quad B = -3\]
\[t^2e^{6t} : \quad 40A = 200 \quad \Rightarrow \quad C = \frac{13}{20}\]

The general solution is,
\[y(t) = c_1 \cos(2t) + c_2 \sin(2t) + \left(5t^2 - 3t + \frac{13}{20}\right)e^{6t}\]

Now apply the initial conditions.
\[-1 = y(0) = c_1 + \frac{13}{20} \quad \Rightarrow \quad c_1 = -\frac{33}{20}\]
\[-3 = y'(0) = 2c_2 + \frac{9}{10} \quad \Rightarrow \quad c_2 = -\frac{39}{20}\]

The actual solution is then,
\[y(t) = -\frac{33}{20} \cos(2t) - \frac{39}{20} \sin(2t) + \left(5t^2 - 3t + \frac{13}{20}\right)e^{6t}\]

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1. Compute the Wronskian.
\[W(y_1, y_2) = \begin{vmatrix} e^{rt} & e^{rt} \\ r_1e^{rt} & r_2e^{rt} \end{vmatrix} = r_2e^{(r_2+r_2)t} - r_1e^{(r_1+r_2)t} = (r_2-r_1)e^{(r_1+r_2)t} \neq 0 \quad \text{b/c} \quad r_2 \neq r_1\]
So they are a fundamental set of solutions and the general solution is what I claimed it to be and the general solution is \(y(t) = c_1e^{rt} + c_2e^{rt}\).

4. I’ll leave it to you to verify that \(y_c(t) = c_1e^{-\frac{9}{38}t} \cos\left(\sqrt{5} t\right) + c_2e^{-\frac{9}{38}t} \sin\left(\sqrt{5} t\right)\). The guess for the particular solution and its derivatives are,
\[Y_p = Ae^9t \quad Y'_p = 9Ae^9t \quad Y''_p = 81e^9t\]
Plugging this into the differential equation and simplifying gives,
\[381A = -4e^{9t}\]
Setting coefficients equal and solving gives : \(381A = -4 \quad \Rightarrow \quad A = \frac{4}{381}\)

The general solution is then : \(y(t) = c_1e^{-\frac{9}{38}t} \cos\left(\sqrt{5} t\right) + c_2e^{-\frac{9}{38}t} \sin\left(\sqrt{5} t\right) - \frac{4}{381}e^{9t}\)

5. I’ll leave it to you to verify that \(y_c(t) = c_1e^{-3t} + c_2e^{3t}\). The guess for the particular solution and its derivatives are,
\[Y_p = A \cos(3t) + B \sin(3t) \quad Y'_p = -3A \sin(3t) + 3B \cos(3t) \quad Y''_p = -9A \cos(3t) - 9B \sin(3t)\]
Plugging this into the differential equation and simplifying gives,
\((-37A - 9B)\cos(3t) + (9A - 37B)\sin(3t) = 20\sin(3t)\)

Setting coefficients equal and solving gives,
\[
\begin{align*}
\cos(3t) & : -37A - 9B = 0 \\
\sin(3t) & : 9A - 37B = 20 \\
& \Rightarrow A = \frac{18}{145} \\
& B = -\frac{74}{145}
\end{align*}
\]

The general solution is then,
\[
y(t) = c_1 e^{-4t} + c_2 e^{7t} + \frac{18}{145} \cos(3t) - \frac{74}{145} \sin(3t)
\]