Step Functions
For problems 1 – 3 find the Laplace transform of the given function.
1. \( f(t) = 13u_{12}(t)e^{\frac{1}{2}t} + 9u_7(t)\sinh(5t - 35) \)

2. \( g(t) = u_2(t)\sin(6t) - 8t^2u_4(t) \)

3. \( h(t) = \begin{cases} 
6t & t < 4 \\
8 + 4t & 4 \leq t < 8 \\
4t + 10e^{-t} & t \geq 8 
\end{cases} \)

For problems 4 – 6 find the inverse transform of each of the following.
4. \( H(s) = \frac{3e^{-s}}{s} + \frac{7e^{-10s}}{s^2 + 16} + \frac{(s + 7)e^{-4s}}{s^2 + 14s + 58} \)

5. \( F(s) = \frac{8 + 4se^{-6s} - 2e^{-10s}}{(s + 3)(s - 7)} \)

6. \( G(s) = \frac{se^{-5s} - 12e^{-4s} + 8se^{-3s}}{s(2s^2 + 8)} \)

IVP’s with Laplace Transforms
Use Laplace transforms to solve the given IVP. In the partial fraction stage all quadratics that can be factored with integer coefficients must be factored!
7. \( y'' - 6y' + 9y = 7e^{3t} \quad y(0) = -8, y'(0) = -1 \)

8. \( 4y'' - 48y' + 145y = 7e^t \quad y(0) = 0, y'(0) = 3 \)

9. \( y'' + 9y = -4\sin(3t) \quad y(0) = 4, y'(0) = -7 \)