IVP's with Step Functions
Use Laplace transforms to solve the given IVP. In the partial fraction stage all quadratics that can be factored with integer coefficients must be factored!

1. \( y'' - 4y' + 20y = 12 - u_4(t) + 8u_6(t) \quad y(0) = 3, \ y'(0) = 1 \)

2. \( 2y'' - y' - 6y = 3u_6(t)e^{2t-12} \quad y(0) = 2, \ y'(0) = -3 \)

3. \( y'' - 4y = 4t - 3(t - 7)u_7(t) \quad y(0) = 0, \ y'(0) = 0 \)

Dirac-Delta Function
Use Laplace transforms to solve the given IVP.

4. \( 9y'' - 30y' + 25y = \delta(t - 2) \quad y(0) = 10, \ y'(0) = 0 \)

5. \( y'' + 11y' + 18y = 6u_6(t) - 8\delta(t - 7) \quad y(0) = 0, \ y'(0) = 0 \)

Convolution Integrals

6. Find the Laplace Transform of \( f(t) = \int_0^t \tau \sin(3\tau - 3t) \sin(9\tau) \, d\tau \).

7. Use a convolution integral (make sure you evaluate the integral!) to find the inverse transform of \( G(s) = \frac{24}{s(s^2 + 64)} \).

8. Find the solution to the following IVP in terms of \( g(t) \).
\[
y'' - 6y' = g(t) \quad y(0) = 0, \ y'(0) = 8
\]