2. (2 pts) Not much to do for the solution.

\[ \int -y^{-3} \, dy = \int 6x^3 + 8x \, dx \quad \Rightarrow \quad \frac{1}{7} y^{-2} = \frac{3}{7} x^4 + 4x^2 + c \]

Now, apply the initial condition to get,

\[ c = \frac{1}{128} \quad \Rightarrow \quad y(x) = \pm \frac{1}{\sqrt[3]{3x^4 + 8x^2 + \frac{1}{64}}} \quad \Rightarrow \quad y(x) = -\frac{1}{\sqrt[3]{3x^4 + 8x^2 + \frac{1}{64}}} \]

Don’t forget to do the +/- when taking the root and reapplying the initial conditions tells us that the “-” is the correct sign.

For the interval of validity it should be fairly clear that the quantity under the root is always positive and so the solution will exist for all x’s and so the interval of validity is: \([-\infty < x < \infty]\).

3. (3 pts) Not much to do for the solution.

\[ \int 2y + 3 \, dy = \int 3 - 8x \, dx \quad \Rightarrow \quad y^2 + 3y = 3x - 4x^2 + c \]

Now, apply the initial condition to get,

\[ 130 = c \quad \Rightarrow \quad y^2 + 3y - (130 + 3x - 4x^2) = 0 \quad \Rightarrow \quad y(x) = \frac{-3 \pm \sqrt{529 + 12x - 16x^2}}{2} \]

Reapplying the initial condition tells us that the actual solution is,

\[ y(x) = \frac{-3 + \sqrt{529 + 12x - 16x^2}}{2} \]

Next, for the interval of validity we need to make sure we don’t take any square roots of negative numbers so, the interval of validity will be (I’ll leave it to you to check the roots work…),

\[-5.3872 < \frac{1}{5} (3 - 5\sqrt{85}) < x < \frac{1}{5} (3 + 5\sqrt{85}) = 6.1372\]

5. (5 pts) Here’s the two IVP’s for the system.

\[ Q'_1 = (6)(5) - \frac{3Q_1}{800 + 2t} = 30 - \frac{3}{2} \frac{Q_1}{400 + t} \quad Q_1(0) = 75 \]

\[ Q'_2 = (3)(4) - \frac{4Q_2}{800 + 2t_c} = 12 - \frac{2}{400 + t_c} Q_2 \quad Q_2(t_c) = 300 \]

Note that at this point we won’t be able to completely give the 2nd IVP until we’ve solved the first and determine when the amount of contaminate reaches 300, \( t_c \).

Here is the solution to the first IVP.

\[ Q_1(t) = 12(400 + t) - \frac{4725(400^3)}{(400 + t)^2} \]
Setting this equal to 300 and solving (which probably requires a computational aid...) yields: \( t = 7.6784 \). So, the second IVP is then,

\[
Q'_2 = 12 - \frac{2}{407.6784}Q_2 = 12 - \frac{1}{203.8392}Q_2 \\
Q_2 (7.6784) = 300
\]

Here is the solution to this IVP.

\[
Q_2 (t) = 2446.0704 - 2228.4524e^{-\frac{t}{203.8392}}
\]

To answer this we then need \( Q_2 (3 + 7.6784) = Q_2 (10.6784) = 331.3535 \). Remember this is 3 hours AFTER the second process started and so will be at \( t = 10.6784 \). Also, if you use the solution from obtained using only 4 decimal places you should get around 277.4692.

**Not Graded**

1. Not much to do here for the solution.

\[
\int e^{4y^2}dy = \int 1 - 2x \, dx \quad \Rightarrow \quad \frac{1}{4}e^{4y^2} = x - x^2 + c
\]

Now, apply the initial condition to get,

\[
c = \frac{1}{4}e^{15} \quad \Rightarrow \quad \frac{1}{4}e^{4y^2} = x - x^2 + \frac{1}{4}e^{15} \quad \Rightarrow \quad y(x) = \frac{1}{2} \left[ 1 + \ln \left( e^{15} + 4x - 4x^2 \right) \right]
\]

For the interval of validity we need to make sure the argument of the logarithms is positive. So,

\[
e^{15} + 4x - 4x^2 = 0 \quad \Rightarrow \quad x = \frac{1}{2} \left( 1 \pm \sqrt{1 + e^{15}} \right) = -903.5213, \ 904.5213
\]

The argument of the logarithm is only positive in \(-903.5213 < x < 904.5213\) and this also contains the initial condition and so we know that this is in fact the interval of validity.

4. Here’s the IVP and the solution (I’ll let you check the solution details).

\[
Q' = 4 \left( 5 + e^{-\frac{t}{30}} \right) - \frac{4Q}{100} \\
Q (0) = 12
\]

\[
Q(t) = 500 - 1088e^{-\frac{t}{25}} + 600e^{-\frac{t}{30}} \quad \lim_{t \to \infty} Q(t) = 500
\]

We can see from the limit that the equilibrium amount of salt would be 500 ounces.

6. Not much to do here other than do a quick set up.

\[
Q'_3 = (12)(0) - \frac{16Q_3}{815.3568 + 4(t - 10.6784)} = -\frac{16Q_3}{772.6432 + 4t} \\
Q_3 (10.6784) = 331.3535
\]

Note that the initial volume of this third case is the same as the initial volume of the second case or,
$800 + 2t_c = 800 + 2(10.6784) = 815.3568$