1. (4 pts) I’ll use weeks as the time frame for my solution(s). Here are the IVPs for this problem.

\[
\begin{align*}
P' &= rP & P(0) &= 200 & P(12) &= 400 \\
P_1' &= rP + 15 - 35 &= rP_1 - 20 & P_1(0) &= 200 \\
P_2' &= rP_2 + 20 - 25 &= rP_2 - 5 & P_2(6) &= P_1(6)
\end{align*}
\]

The solution to the first one (so we can find \( r \)) is,

\[
P(t) = 200e^{rt}
\]

so

\[
400 = 200e^{12r} \quad \Rightarrow \quad r = \frac{1}{12} \ln (2)
\]

The solution (I’ll leave the details to you to check) to the second IVP (for the time frame \( 0 \leq t \leq 6 \)) is,

\[
P_1(t) = \frac{240}{11} - 146.2468e^{\frac{1}{12}t} \quad P_1(6) = 139.4226
\]

The solution to the third IVP (for the time \( t \geq 6 \)) is,

\[
P_2(t) = \frac{60}{11} + 37.3783e^{\frac{1}{12}t} \quad P_2(12) = 161.3183
\]

So, after 3 months there are only 161 (or so....) mice left.

2. (3 pts) Here’s the IVP for this situation.

\[
v' = 9.8 - \frac{12}{8} v, \quad v(0) = 1.5
\]

The solution to this IVP and the height function (using \( s_1(0) = 0 \) as an initial condition is,

\[
v_1(t) = \frac{98}{15} - \frac{151}{30} e^{-1.2t} \quad s_1(t) = \int v(t) dt = \frac{98}{15} t + \frac{151}{45} e^{-1.2t} + c = \frac{98}{15} t + \frac{151}{45} e^{-1.2t} - \frac{151}{45}
\]

After 3 seconds both the velocity (since we’ll need this for the next problem) and the height (which gives us the answer we need for this problem are,

\[
v_1(3) = 6.4774 \quad s_1(3) = 16.2817
\]

So, the bridge was 16.2817 meters above the lake.

4. (3 pts) It looks like the equilibrium solutions for this differential equation are,

\[
y = -3 \quad y = 3 \quad y = 5
\]

A sketch of some solutions is to the right and from this we can see that,

\[
y = -3 \quad \text{asymptotically stable} \\
y = 3 \quad \text{unstable} \\
y = 5 \quad \text{asymptotically stable}
\]
3. Here’s the IVP for this situation.

\[ v_2' = 9.8 - \frac{30}{8} v_2' = 9.8 - \frac{15}{2} v_2 \]

\[ v_2 (3) = 6.4774 \]

The solution to this IVP and the height function (using \( s_2 (3) = 16.2817 \) as an initial condition is,

\[ v_2 (t) = \frac{196}{75} + 297069.1353 e^{-\frac{15}{2}t} \]

\[ s_2 (t) = \int v_2 (t) dt = \frac{196}{75} t - 79218.4361 e^{-\frac{15}{2}t} + c = \frac{196}{75} t - 79218.4361 e^{-\frac{15}{2}t} + 9.4721 \]

Now, in order to determine the velocity of the object when it hits the bottom of the lake we need to determine when it hits the bottom. To do this we’ll need to solve,

\[ s_2 (t) = 16.2817 + 30 = 46.2817 \quad \Rightarrow \quad t = 14.0853 \]

Remember that we’re measuring zero from the bridge and so we need to take that into account.

So, when the object hits the bottom it is going \( v_2 (14.0853) = 2.6133 \text{ m/s} \).

5. It looks like the equilibrium solutions for this differential equation are,

\[ y = 0 \quad y = 3 \]

A sketch of some solutions is to the right and from this we can see that,

\[ y = 0 \quad \text{unstable} \]

\[ y = 3 \quad \text{semi-stable} \]

6. We just need to run through the formulas using \( f(t, y) = t y e^{-y}, \quad t_0 = 2 \) and \( y_0 = 2.5 \) Here’s the results for \( h = 0.5 \).

<table>
<thead>
<tr>
<th>( t )</th>
<th>( 2.5 )</th>
<th>( 3 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( f_n )</td>
<td>3.0326532986</td>
<td>2.2041276315</td>
</tr>
<tr>
<td>Approx.</td>
<td>4.0163266493</td>
<td>5.1183904650</td>
</tr>
</tbody>
</table>

Here’s the results for \( h = 0.25 \).

| \( t \) | \( 1.25 \) | \( 2.5 \) | \( 2.75 \) | \( 3 \) |
|-------|-------|-------|-------|
| \( f_n \) | 3.0326532986 | 2.6749495780 | 2.3566485561 | 2.1237398584 |
| Approx. | 3.2581633246 | 3.9269007191 | 4.5160628582 | 5.0469978228 |

For \( h = 0.5 \) we have \( y(3) \approx 5.1183904650 \) and for \( h = 0.25 \) we have \( y(3) \approx 5.0469978228 \).