#1. (2 pts) This is the function we used in #13 from the first homework set so all we need use the work from the last homework set and do the integral.

\[ L = \int_0^4 [v'(t)]^2 \, dt = \int_0^4 13e^{2t} \, dt = \left[ \frac{13}{2} e^{2t} \right]_0^4 = \frac{13}{2} (e^8 - 1) \]

#4. (2 pts) This function is not continuous at (0,0) so let’s see if we can find a couple of paths that give different values for the limit.

For the y-axis \((x = 0)\):

\[ \lim_{(x,y) \to (0,0)} \frac{x^6 y}{3y^2 - x^{12}} = \lim_{(x,y) \to (0,0)} \frac{0}{3y^2} = 0 \]

For \(y = x^6\):

\[ \lim_{(x,y) \to (0,0)} \frac{x^6 y}{3y^2 - x^{12}} = \lim_{(x,y) \to (0,0)} \frac{x^6 x^6}{3x^{12} - x^{12}} = \lim_{(x,y) \to (0,0)} \frac{1}{2} = \frac{1}{2} \]

So, we have two paths that give different values of the limit and so we know that \(\lim_{(x,y) \to (0,0)} \frac{x^4 y^2}{3y^3 - x^{12}}\) doesn’t exist.

#6. (2 pts)

\[ w_x = \cos(x - z)e^{2x-y^2} + 2 \sin(x - z)e^{2x-y^2} \]

\[ w_y = -2y \sin(x - z)e^{2x-y^2} \]

\[ w_z = -\cos(x - z)e^{2x-y^2} \]

#10. (2 pts)

\[ h_u = 2uv^2e^{u^2v^2} + \frac{3v}{u^2} \]

\[ h_v = 2u^2ve^{u^2v^2} - \frac{3}{u} \]

\[ h_{uv} = 2v^2e^{u^2v^2} + 4u^2v^4e^{u^2v^2} - \frac{6v}{u^3} \]

\[ h_{vv} = 4uve^{u^2v^2} + 4u^3v^3e^{u^2v^2} + \frac{3}{u^2} \]

#11. (2 pts) Because we can do the integrals in any order we’ll do them in a different order than that requested to make our life a little easier.

\[ f_t = \frac{2t}{(u + t^2)^{10}} \]

\[ f_u = 2\cos(u + t^2) - 4t^2 \sin(u + t^2) \]

\[ f_{tu} = -2\sin(u + t^2) - 4t^2 \cos(u + t^2) \]

\[ f_{tua} = -2\cos(u + t^2) + 4t^2 \sin(u + t^2) = f_{atu} \]
#2. Not much to do here. This function is continuous at (0,0) so:
\[
\lim_{(x,y)\to(0,0)} \frac{e^{x^2-y^2}}{3x+7y-5} = \frac{1}{5}
\]

#3. This function is not continuous at (0,0) so let’s see if we can find a couple of paths that give different values for the limit.

\textbf{x-axis (} y = 0 \textbf{): }
\[
\lim_{(x,y)\to(0,0)} \frac{(y-3x)^3}{2y^3+5x^3} = \lim_{(x,y)\to(0,0)} \frac{(-3x)^3}{5x^3} = \lim_{(x,y)\to(0,0)} \frac{-27}{5} = -\frac{27}{5}
\]

\textbf{y-axis (} x = 0 \textbf{): }
\[
\lim_{(x,y)\to(0,0)} \frac{(y-3x)^3}{2y^3+5x^3} = \lim_{(x,y)\to(0,0)} \frac{(y)^3}{2y^3} = \lim_{(x,y)\to(0,0)} \frac{1}{2} = \frac{1}{2}
\]

So, we have two paths that give different values of the limit and so we know that \(\lim_{(x,y)\to(0,0)} \frac{(y-3x)^3}{2y^3+5x^3}\) doesn’t exist.

#5. \(g_x = 2xyz - \frac{2}{2x-3z} - 12 \cos^2(4x) \sin(4x)\) \quad \(g_y = x^2z^3\) \quad \(g_z = 3x^2yz^2 + \frac{3}{2x-3z}\)

#7.

\[
f_u = \frac{s^2}{u} + 2u \sec^2(u^2 + 4s) \quad f_v = -\frac{s^2}{v}
\]

\[
f_s = 2s \ln\left(\frac{4u}{v}\right) + 4 \sec^2(u^2 + 4s) \quad f_t = 0
\]

#8. First let’s find \(\frac{\partial z}{\partial x}\)

\[
2z \frac{\partial z}{\partial x} = 2xe^{3+4z} + 4x^2e^{3+4z} \frac{\partial z}{\partial x}
\]

\[
(2z - 4x^2e^{3+4z}) \frac{\partial z}{\partial x} = 2xe^{3+4z} \quad \Rightarrow \quad \frac{\partial z}{\partial x} = \frac{xe^{3+4z}}{2z - 2xe^{3+4z}}
\]

Now \(\frac{\partial z}{\partial y}\)

\[
2z \frac{\partial z}{\partial y} + 3 \sec(3y) \tan(3y) = 4x^2e^{3+4z} \frac{\partial z}{\partial y}
\]

\[
3 \sec(3y) \tan(3y) = (4x^2e^{3+4z} - 2z) \frac{\partial z}{\partial y} \quad \Rightarrow \quad \frac{\partial z}{\partial y} = \frac{3 \sec(3y) \tan(3y)}{4x^2e^{3+4z} - 2z}
\]
#9.

\[ z_x = 1 - \frac{1}{x} \quad z_y = 12y^3 - \frac{1}{y} \]

\[ z_{xx} = \frac{1}{x^2} \quad z_{xy} = 0 \quad z_{yx} = 0 \quad z_{yy} = 36y^2 + \frac{1}{y^2} \]

#12. Order probably won’t matter much here so we’ll do the derivatives in the requested order.

\[
\frac{\partial u}{\partial x} = \frac{2}{3} z^4 y^{-2} x^{-\frac{1}{3}} - \frac{1}{x} \\
\frac{\partial^2 u}{\partial y \partial x} = -\frac{4}{3} z^4 y^{-3} x^{-\frac{1}{3}} \\
\frac{\partial^3 u}{\partial y^2 \partial x} = 4z^4 y^{-4} x^{-\frac{1}{3}} \\
\frac{\partial^4 u}{\partial z \partial y^2 \partial x} = 16z^3 y^{-4} x^{-\frac{1}{3}} \\
\frac{\partial^5 u}{\partial x \partial z \partial y^2 \partial x} = -\frac{16}{3} z^3 y^{-4} x^{-\frac{2}{3}}
\]