Chain Rule

1. Use the Chain Rule to find $\frac{dz}{dq}$ given that,

$$z = v^2 - \left(x^2 + 3y^2\right) \qquad x = \mathbf{e}^{-4q} \qquad y = q^3 \qquad v = \cos(5q)$$

2. Use the Chain Rule to find $\frac{dz}{dx}$ given that $z = \tan\left(x^2 + 2y\right)$, $y = \mathbf{e}^{x^3}$

3. Use the Chain Rule to find formulas for $\frac{\partial w}{\partial s}$ and $\frac{\partial w}{\partial t}$ given that,

$$w = w(x, y, z) \qquad x = x(s, v) \qquad y = y(p, v) \qquad z = z(p) \qquad v = (s, t) \qquad p = p(s)$$

4. Use the Chain Rule to find $\frac{dy}{dx}$ for $\sin\left(xy^2\right) + 3x = 2 + y^3$

5. Use the Chain Rule to find $\frac{\partial z}{\partial x}$ and $\frac{\partial z}{\partial y}$ for $z^2 + \sec(3y) = x^2\mathbf{e}^{3+4z}$.

Directional Derivatives

6. Find ∇f and the directional derivative for $f(x, y) = x\mathbf{e}^{x+5y}$ in the direction of $\vec{v} = \langle 3, -2\rangle$ at the point $(5, -1)$.

7. Find the directional derivative of $f(x, y, z) = z^2\sin(x) - \frac{x^2}{y^3}$ in the direction of $\vec{v} = \langle 2, -1, -6\rangle$.

8. Find the maximum rate of change of $f(x, y, z) = \frac{3z}{y} - \frac{y}{2x}$ at the point $(1, -4, 0)$ and the direction in which it occurs.

9. Given that $\vec{u} = \left\langle \frac{4}{5}, -\frac{3}{5} \right\rangle$, $\vec{v} = \left\langle \frac{2}{\sqrt{13}}, \frac{3}{\sqrt{13}} \right\rangle$, $\vec{w} = \left\langle -\frac{1}{\sqrt{17}}, \frac{4}{\sqrt{17}} \right\rangle$, $D_{\vec{u}} f(2,1) = -6$ and $D_{\vec{v}} f(2,1) = \frac{12}{\sqrt{13}}$ determine the value of $D_{\vec{w}} f(2,1)$.