Double Integrals in Polar Coordinates
For problems 1 & 2 evaluate the integral over the given region.
1. \[ \iint_D 6x^2y \, dA, \quad D \text{ is the region between } x^2 + y^2 = 9 \text{ and } x^2 + y^2 = 25 \text{ and to the left of the } y\text{-axis.} \]

2. \[ \iint_D e^{-3x^2-3y^2} \, dA, \quad D \text{ is the disk of radius 6 centered at the origin.} \]

3. Find the volume of the solid that is bounded by \( y = 16 - x^2 - z^2 \) and \( y = 2x^2 + 2z^2 - 32 \). Note that you will have to use a modified version of polar coordinates to do this problem.

4. Use a double integral to derive the formula for the area of a circle of radius \( a \).

5. Evaluate \[ \iint_{-3}^{0} \cos \left( 2x^2 + 2y^2 \right) \, dy \, dx \] by converting the integral into polar coordinates.

Triple Integrals
For problems 6 – 9 evaluate the given integral.

6. \[ \iiint_1^2 \int_0^6 \int_0^{x^2 \sin \left( \frac{2}{y} \right)} \, dx \, dz \, dy \]

7. \( \iiint_E 3 - 12z \, dV \) where \( E \) is the solid bounded by the planes \( x + 2y + 2z = 6 \), \( x = 0 \), \( y = 0 \), and \( z = 0 \). In other words \( E \) is the solid that lies beneath \( 2x + y + 3z = 6 \) and in the first octant.

8. \( \iiint_E y \, dV \) where \( E \) is the solid that lies between \( x + 2y + 2z = 6 \) and \( 2x + 4y + 4z = 20 \) and is in front of the triangle in the \( yz\)-plane with vertices (0,0), (1,0) and (1,2) – these are in the form \( (y,z) \).

9. \( \iiint_E \sqrt{y^2 + z^2} \, dV \) where \( E \) is the solid that is in front of \( x = 4y^2 + 4z^2 - 3 \) and behind \( x = 1 \).

10. Use a triple integral to find the volume of the solid \( E \) used in problem 8.

Triple Integrals with Cylindrical Coordinates
For problems 11 – 13 you must use cylindrical coordinates to do the problem.

11. \( \iiint_E y \, dV \) where \( E \) is the solid that lies inside \( x^2 + y^2 = 9 \), above \( z = -\frac{1}{2} x^2 - \frac{1}{2} y^2 \) and below \( z = \sqrt{x^2 + y^2} \)

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12. Find the volume of the solid $E$ that is bounded by $y = x^2 + z^2 - 6$ and $y = 9 - 2x^2 - 2z^2$.

13. Use a triple integral to find a formula for the volume of a cylinder of radius $a$ and height $h$. 