Math 3435 **Homework Set 7** **10 Points**

Triple Integrals with Spherical Coordinates

For problems 1 and 2 you must use spherical coordinates to do the problems.

1. $\iiint\limits_E z^2\,dV$ where *E* is the region below the sphere of radius 2 and inside the cone $\varphi = \dfrac{\pi}{6}$.

2. $\iiint\limits_E \left(x^2+y^2+z^2\right)^{\frac{3}{2}}\,dV$ where *E* is the region that lies in the first octant and between the spheres of radius 2 and 5.

3. Evaluate the following integral by first converting to spherical coordinates.

$$\int_{-2}^{2}\int_{-\sqrt{4-y^2}}^{0}\int_{-\sqrt{16-x^2-y^2}}^{-\sqrt{3x^2+3y^2}} x\,dz\,dx\,dy$$

Change of Variables

For problems 4 and 5 find the Jacobian of the transformation.

4. $x = 2u^2+4v^2-5u, \quad y = u^2 v$

5. $x = \mu\cos\alpha, \ y = \mu\sin\alpha$

For problems 6 – 8 find and graph the image of the set *R* under the given transformation. Note that the point of these examples is not necessarily to transform *R* into a "nice" region. Instead all we're trying to do is apply some transformations to regions.

6. *R* is the triangle with vertices (0,0), (2,4), (2,-8) and the transformation is $x = \sqrt{3+v}, \ y = \frac{1}{2}u$.

Note that, depending on how you write your equations for each side, you may find it useful to use the ranges of possible *y* values to in order to get a range of possible *u* values.

7. *R* is the rectangle bounded by the lines $x = 0, \ x = 4, \ y = 0, \ y = 1$ and the transformation is $u = 4y^2 - x^2, \ v = 2xy$.

Sometimes transformations are giving "backwards" as they are here. In these cases it is usually best to plug the equations defining *R* into the transformations rather than plugging the transformations in the equations defining *R* as we've done to this point. Also, you will want ranges for *u* and/or *v* on at least a couple of the "new" equations.

8. *R* is the disk given by $x^2+y^2 \le 1$ and the transformation is $u = ax, \ v = by$.

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9. If *R* is the parallelogram with vertices (0,0), (8,4), (10,10) and (2,6) use the transformation $x = \frac{2}{5}\left(v - u + 1\right)$ $y = \frac{1}{5}\left(6v - u + 6\right)$ to convert the region and evaluate the integral $\iint\limits_R 10x + 5y\,dA$.

10. If *R* is the ellipse $\frac{x^2}{16} + 9y^2 = 1$ determine a transformation that will convert this into a circle of radius 1 and use that transformation to evaluate $\iint\limits_R x^2\,dA$.

Note that you've already seen how to turn a circle of radius 1 into an ellipse so use that as a guide to determine this transformation.

<u>Surface Area</u>

For problems 11 and 12 find the area of the given surface.

11. The part of the plane $6x + 2y + 5z = 10$ that lies in the first octant.

12. The part of the surface $z = 3 + 4x + 6y^2$ that lies above the triangle in the *xy*-plane with vertices (0,0), (10,5) and (0,5).

13. In class I have you the formula for the surface area of $z = f\left(x, y\right)$ that lies above a region *D* in the *xy*-plane. Use a modification of this formula to find the surface area of the portion of $x = 4y^2 + 4z^2 - 10$ that lies behind $x = 2$.