Math 3435 **Homework Set 9** **10 Points**

Line Integrals of Vector Fields

For problems 1 and 2 evaluate $\int_C \vec{F} \bullet d\vec{r}$ for the given vector field, $\vec{F}$, and the given curve, *C*.

1. $\vec{F}(x,y) = y\vec{i} + (y-x)\vec{j}$ where *C* is the upper half of $x^2 + \frac{y^2}{9} = 1$ oriented in the clockwise direction.

2. $\vec{F}(x,y,z) = x\mathbf{e}^{2y}\vec{i} - yz^2\vec{j} + (x-3y)\vec{k}$ where *C* is given by $\vec{r}(t) = 4t\vec{i} + (2+t^2)\vec{j} - 3t^2\vec{k}$, $0 \le t \le 2$

Fundamental Theorem for Line Integrals

3. Evaluate $\int_C \nabla f \bullet d\vec{r}$ for the function $f(x,y,z) = (z + 2y - x^2)\cos(\pi xy)$ and *C* is given by $\vec{r}(t) = \langle 6-t^2, 4t, t^3 - t^2 \rangle$, $1 \le t \le 2$.

4. For the vector field from **#1** above is it possible to determine if $\int_C \vec{F} \bullet d\vec{r}$ is independent of path? If so, is the integral independent of path and if it is not possible explain why not.

Conservative Vector Fields

For problems 5 and 6 determine if the vector field, $\vec{F}$, is conservative or not. If it is conservative find the potential function for the vector field.

5. $\vec{F} = (4x^2 - xy^3)\vec{i} + (8xy + x^2y^2)\vec{j}$

6. $\vec{F} = (9x^2y - 8x - 4xy^4)\vec{i} - (8x^2y^3 - 3x^3 - 7)\vec{j}$

For problems 7 – 9 find the potential function for the vector field and then evaluate $\int_C \vec{F} \bullet d\vec{r}$ for the given curve *C*.

7. $\vec{F} = (9x^2y - 8x - 4xy^4)\vec{i} - (8x^2y^3 - 3x^3 - 7)\vec{j}$ and *C* is the line segment from (5,-1) to (3,0).

8. $\vec{F} = (2\sin(4y)\mathbf{e}^{2x+3y} - 3x^2)\vec{i} + (4\cos(4y)\mathbf{e}^{2x+3y} + 3\sin(4y)\mathbf{e}^{2x+3y})\vec{j}$ and *C* is the left half of the circle of radius 8 that is centered at (10,4) starting at the bottom of the circle and ending at the top.

9. $\vec{F} = (2x\ln(z) - \sqrt{y} - 9z^3)\vec{i} - \left(\frac{x-2}{2\sqrt{y}}\right)\vec{j} + \left(\frac{x^2}{z} - 27xz^2\right)\vec{k}$ and *C* is $\vec{r}(t) = t^4\vec{i} + 3\vec{j} - (1-t^2)\vec{k}$, $\sqrt{2} \le t \le 2$.

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Green's Theorem

For problems 10 and 11 evaluate the line integral (a) directly and (b) using Green's Theorem. Assume all curves have the positive orientation.

10. $\oint_C \left(8y + 2xy^2\right) dx + \left(2x^2 y - 8x\right) dy$ where *C* is the circle of radius 1 centered at the origin.

11. $\oint_C xy\, dy - \left(1 - y^2\right) dx$ where *C* is the triangle with vertices (0,0), (-2,2) and (4,2).