Green’s Theorem
For problems 1 and 2 sketch the positively oriented curve (clearly indicating the positive orientation) and use Green’s Theorem to evaluate the line integral along the given curve.

1. \( \oint_C 2xy \, dy - (1 - 3y) \, dx \) where \( C \) is the portion of \( y = x^3 \) from (-1,-1) to (2,8) and the three line segments: (i) from (2,8) to (2,-4), (ii) from (2,-4) to (-1,-4), (iii) from (-1,-4) to (-1,-1).

2. \( \int_C \mathbf{F} \cdot d\mathbf{r} \) where \( \mathbf{F} = (y^3 + x^4) \mathbf{i} + (x^2 - x^3) \mathbf{j} \) and \( C \) is the portion of \( x^2 + y^2 = 4 \) from \((-\sqrt{3},-1)\) to \((-2,0)\) in a counter-clockwise direction and the two line segments: (i) from \((-2,0)\) to \((0,0)\) and (ii) \((0,0)\) to \((-\sqrt{3},-1)\).

Curl and Divergence

3. Find the curl and divergence of \( \mathbf{F} = \frac{yz}{x^2} \mathbf{i} + z \cos(x) \mathbf{j} + (3x - 4y + z^2) \mathbf{k} \)

For problems 4 and 5 use the curl to determine if the given vector field is conservative or not.

4. \( \mathbf{F} = \frac{yz}{x^2} \mathbf{i} + z \cos(x) \mathbf{j} + (3x - 4y + z^2) \mathbf{k} \)

5. \( \mathbf{F} = \left(2x \ln(z) - \sqrt{y - 9z^3}\right) \mathbf{i} - \left(\frac{x-2}{2\sqrt{y}}\right) \mathbf{j} + \left(\frac{x^2}{z} - 27xz^2\right) \mathbf{k} \)

Parametric Surfaces
For problems 6 – 9 find a parametric representation for the given surface.

6. The plane containing the points \((0,3,-5)\), \((3,0,-4)\) and \((1,-6,0)\).

7. The portion of \( z = 4 - 10x^2 - 10y^2 \) that lies above \( z = -1 \).

8. The cylinder \( x^2 + z^2 = 7 \) between \( y = -4 \) and \( y = -1 \).

9. The lower half of the sphere \( x^2 + y^2 + z^2 = 9 \).

10. Find the tangent plane to \( x = u(\nu - 2), \ y = 4\nu^2, \ z = u^2 - 3 \) at the point \((-4,16,-2)\).

For problems 11 and 12 find the area of the given surface.

11. The portion of \( z = 4 + x + 3y^2 \) that lies above the triangle with vertices \((0,0)\), \((-6,3)\) and \((0,3)\).

12. The surface \( r(u,v) = 6uv \mathbf{i} - v \mathbf{j} + u \mathbf{k} \) where \( u^2 + v^2 \leq 16 \).