Math 3435 **Homework Set 10** **10 Points**

Green's Theorem

For problems 1 and 2 sketch the positively oriented curve (clearly indicating the positive orientation) and use Green's Theorem to evaluate the line integral along the given curve.

1. $\oint_C 2xy\,dy - (1-3y)\,dx$ where *C* is the portion of $y = x^3$ from (-1,-1) to (2,8) and the three line segments : (*i*) from (2,8) to (2,-4), (*ii*) from (2,-4) to (-1,-4), (*iii*) from (-1,-4) to (-1,-1) .

2. $\int_C \vec{F} \bullet d\vec{r}$ where $\vec{F} = \left(y^3 + x^4\right)\vec{i} + \left(x^2 - x^3\right)\vec{j}$ and *C* is the portion of $x^2 + y^2 = 4$ from $\left(\sqrt{3},-1\right)$ to (-2,0) in a counter-clock wise direction and the two line segments : (i) from (-2,0) to (0,0) and (*ii*) (0,0) to $\left(\sqrt{3},-1\right)$.

Curl and Diverence

3. Find the curl and divergence of $\vec{F} = \frac{zy}{x^2}\vec{i} + z\cos(x)\,\vec{j} + \left(3x - 4y + z^2\right)\vec{k}$

For problems 4 and 5 use the curl to determine if the given vector field is conservative or not.

4. $\vec{F} = \frac{zy}{x^2}\vec{i} + z\cos(x)\,\vec{j} + \left(3x - 4y + z^2\right)\vec{k}$

5. $\vec{F} = \left(2x\ln(z) - \sqrt{y} - 9z^3\right)\vec{i} - \left(\frac{x-2}{2\sqrt{y}}\right)\vec{j} + \left(\frac{x^2}{z} - 27xz^2\right)\vec{k}$

Parametric Surfaces

For problems 6 – 9 find a parametric representation for the given surface.

6. The plane containing the points (0,3,-5), (3,0,-4) and (1,-6,0).

7. The portion of $z = 4 - 10x^2 - 10y^2$ that lies above $z = -1$.

8. The cylinder $x^2 + z^2 = 7$ between $y = -4$ and $y = -1$.

9. The lower half of the sphere $x^2 + y^2 + z^2 = 9$.

10. Find the tangent plane to $x = u(uv - 2)$, $y = 4v^2$, $z = u^2 - 3$ at the point (-4,16,-2).

For problems 11 and 12 find the area of the given surface.

11. The portion of $z = 4 + x + 3y^2$ that lies above the triangle with vertices (0,0), (-6,3) and (0,3).

12. The surface $r(u,v) = 6uv\,\vec{i} - v\,\vec{j} + u\,\vec{k}$ where $u^2 + v^2 \le 16$.