4. (2 pts) From our calculator we get : $\sin(2x)=\frac{1}{3} \quad \Rightarrow \quad 2x=\sin^{-1}\left(\frac{1}{3}\right)=0.3398$. From a quick sketch of a unit circle we can see that the second angle will be $\pi-0.3398=2.8018$. Now, all solutions are,

$$\begin{aligned} 2x &= 0.3398+2\pi n \\ 2x &= 2.8018+2\pi n \end{aligned} \qquad \Rightarrow \qquad \boxed{\begin{aligned} x &= 0.1699+\pi n \\ x &= 1.4009+\pi n \end{aligned} \qquad n=0,\pm1,\pm2,\ldots}$$

Plugging in values of *n* gives the following solutions.

$$\begin{array}{llll} n=-2: & \cancel{x=-6.1133}<-5 & \text{OR} & -4.8823 \\ n=-1: & x=-2.9717 & \text{OR} & x=-1.7407 \\ n=0: & x=0.1699 & \text{OR} & \cancel{x=1.4099}>1 \end{array}$$

So, we have the above four solutions.

6. (2 pts) Don't forget that you can't cancel terms from both sides of an equation when solving unless you know that they won't be zero.

$$x^2-3-3\left(x^2-3\right)\mathbf{e}^{7x-2}=\left(x^2-3\right)\left(1-3\mathbf{e}^{7x-2}\right)=0$$

So, from this point we see that,

$$x^2-3=0 \quad \rightarrow \quad x^2=3 \quad \rightarrow \quad \boxed{x=\pm\sqrt{3}}$$

$$1-3\mathbf{e}^{7x-2}=0 \quad \rightarrow \quad \mathbf{e}^{7x-2}=\tfrac{1}{3} \quad \rightarrow \quad 7x-2=\ln\left(\tfrac{1}{3}\right) \quad \rightarrow \quad \boxed{x=\tfrac{1}{7}\left(2+\ln\left(\tfrac{1}{3}\right)\right)=0.1288}$$

7. (2 pts) First we need to combine the logarithms and then exponentiate both sides.

$$\ln\left[(2x+1)(2-x)\right]=\ln\left[-2x^2+3x+2\right]=-1 \quad \rightarrow \quad -2x^2+3x+2=\mathbf{e}^{-1}$$

So, we have a quadratic equation, $-2x^2+3x+2-\mathbf{e}^{-1}=0$ and we'll need to use the quadratic equation to find the solutions.

$$x=\frac{-3\pm\sqrt{9-4(-2)\left(2-\mathbf{e}^{-1}\right)}}{2(-2)}=\boxed{\frac{-3\pm\sqrt{9+8\left(2-\mathbf{e}^{-1}\right)}}{-4}=-0.4241,\quad 1.92412}$$

Checking in the original equation we can see that **IN THIS CASE** both of these values are solutions.

8. (2 pts) The average velocity is,

$$AV=\frac{s(t)-s(2)}{t-2}=\frac{\mathbf{e}^{t^2-4}-t^2-(-3)}{t-2}=\frac{\mathbf{e}^{t^2-4}-t^2+3}{t-2}$$

Here is the table of values for the given values of *t*.

t	*AV*	*t*	*AV*
2.1	-0.67056874	1.9	0.96817785
2.01	-0.07855229	1.99	0.08148605
2.001	-0.00798535	1.999	0.00801469
2.0001	-0.00079985	1.9999	0.00080015

So, from these tables it looks like the average velocities are approaching zero. Therefore we can estimate that the instantaneous velocity is zero and so the object will **not be moving** at $t=2$.

12. (2 pts) Here is the table of values.

t	$f(x)$	*T*	$f(x)$
0.9	1.78370613	1.1	2.23378969
0.99	1.97758746	1.01	2.02258754
0.999	1.99775087	1.001	2.00225088
0.9999	1.99977501	1.0001	2.00022501

So, from the values of this table it looks like we can estimate,

$$\lim_{t\to 1}\frac{t^4-\cos(t-1)}{t^2-1}=2$$

Not Graded

1. The first solution (from our calculator) is : $\cos\left(\frac{t}{3}\right)=-\frac{1}{4} \quad\Rightarrow\quad \frac{t}{3}=\cos^{-1}\left(-\frac{1}{4}\right)=1.8235$. From a quick sketch of a unit circle we can see that the second solution is $\frac{t}{3}=2\pi-1.8235=4.4597$. All solutions are then,

$$\begin{aligned}\tfrac{t}{3}&=1.8235+2\pi n\\ \tfrac{t}{3}&=4.4597+2\pi n\end{aligned} \qquad\Rightarrow\qquad \boxed{\begin{aligned}t&=5.4705+6\pi n\\ t&=13.3791+6\pi n\end{aligned}\qquad n=0,\pm1,\pm2,\ldots}$$

2. All we need to do here is plug in values of *n* to find the answers in the interval.

$$\begin{array}{llll} n=-1: & t=-13.3791 & \text{OR} & t=-5.4705\\ n=0: & t=5.4705 & \text{OR} & t=13.3791\\ n=1: & t=24.3201 & \text{OR} & \cancel{t=32.2287}>25\end{array}$$

So, we have the five solutions above. Note that we know that we didn't need to go past $n=-2$ because that would have meant subtracting another $6\pi=18.8496$ off which would have clearly put the values outside of the interval.

3. From our calculator we get : $\sin(6x)=-\frac{4}{7} \quad\Rightarrow\quad 6x=\sin^{-1}\left(-\frac{4}{7}\right)=-0.6082$. From a quick sketch of a unit circle we can see that a positive angle corresponding to this is $2\pi-0.6082=5.6750$. You don't need to use this but I will for these solutions. We can also see from the unit circle that the second angle will be $\pi+0.6082=3.7498$. Now, all solutions are,

$$\begin{aligned}6x&=3.7498+2\pi n\\ 6x&=5.6750+2\pi n\end{aligned} \qquad\Rightarrow\qquad \boxed{\begin{aligned}x&=0.6250+\tfrac{\pi n}{3}\\ x&=0.9458+\tfrac{\pi n}{3}\end{aligned}\qquad n=0,\pm1,\pm2,\ldots}$$

Plugging in values of *n* gives the following solutions.

$$n=0: \quad x=0.6250 \qquad \text{OR} \qquad x=0.9458$$
$$n=1: \quad x=1.6722 \qquad \text{OR} \qquad x=1.9930$$

So, we have the above four solutions.

5. Not much to this one.

$$\mathbf{e}^{1-x^2}=\tfrac{3}{10} \quad \to \quad 1-x^2=\ln\left(\tfrac{3}{10}\right) \quad \to \quad x^2=1-\ln\left(\tfrac{3}{10}\right) \quad \to \quad \boxed{x=\pm\sqrt{1-\ln\left(\tfrac{3}{10}\right)}=\pm 1.4846}$$

9. The slopes of the secant lines are,

$$m_{PQ}=\frac{f(x)-f(0)}{x-0}=\frac{\dfrac{\sqrt{x^2+4}}{x+1}-2}{x}$$

x	m_{PQ}	*x*	m_{PQ}
-0.1	-2.24998266	0.1	-1.79546873
-0.01	-2.02272726	0.01	-1.97772279
-0.001	-2.00225225	0.001	-1.99775225
-0.0001	-2.00022502	0.0001	-1.99977502

It looks like the tangents of the secant lines are moving towards -2 and so the tangent line is then,

$$\boxed{y=f(0)+m(x-0)=2-2x}$$

10. To say $\lim\limits_{x\to 7} f(x)=-25$ we mean that as we let *x* approach the value of 7, from both the left and the right, the function, $f(x)$, is getting closer and closer to the value of -25. Also, it is completely possible to have $f(7)=100$ because we know that the limit does not care about the that function is doing at $x=7$ and it only cares about what the function is doing around the point. Therefore the function does NOT have to have the same value as the limit.

11. Here is the table of values.

x	$f(x)$	*x*	$f(x)$
-2.9	-9.9	-3.1	-10.1
-2.99	-9.99	-3.01	-10.01
-2.999	-9.999	-3.001	-10.001
-2.9999	-9.9999	-3.0001	-10.0001

So, from the values of this table it looks like we can estimate,

$$\lim_{x\to -3}\frac{x^2-4x-21}{x+3}=-10$$