1. (2 pts)

(a)

$f(-1)=1 \qquad \lim\limits_{x\to -1^-} f(x)=1 \qquad \lim\limits_{x\to -1^+} f(x)=-2$

$\lim\limits_{x\to -1} f(x)=\text{doesn't exist b/c } \lim\limits_{x\to -1^-} f(x)\neq \lim\limits_{x\to -1^+} f(x)$

(b) $f(1)=\text{doesn't exist} \qquad \lim\limits_{x\to 1^-} f(x)=2 \qquad \lim\limits_{x\to 1^+} f(x)=2 \qquad \lim\limits_{x\to 2} f(x)=2$

(c) $f(4)=-2 \qquad \lim\limits_{x\to 4^-} f(x)=-3 \qquad \lim\limits_{x\to 4^+} f(x)=-3 \qquad \lim\limits_{x\to 4} f(x)=-3$

4. (2 pts) $\lim\limits_{t\to -3}\dfrac{(t-6)(t+5)+15-t}{t^2+9t+18}=\lim\limits_{t\to -3}\dfrac{t^2-2t-15}{t^2+9t+18}=\lim\limits_{t\to -3}\dfrac{(t-5)(t+3)}{(t+6)(t-3)}=\lim\limits_{t\to -3}\dfrac{t-5}{t+6}=\boxed{-\dfrac{8}{3}}$

5. (2 pts)

$$\lim_{z\to 4}\frac{\sqrt{z^2-12}-2}{4-z}=\lim_{z\to 4}\frac{\left(\sqrt{z^2-12}-2\right)}{(4-z)}\frac{\left(\sqrt{z^2-12}+2\right)}{\left(\sqrt{z^2-12}+2\right)}=\lim_{z\to 4}\frac{z^2-12-4}{(4-z)\left(\sqrt{z^2-12}+2\right)}$$

$$=\lim_{z\to 4}\frac{(z-4)(z+4)}{-(z-4)\left(\sqrt{z^2-12}+2\right)}=\lim_{z\to 4}\frac{z+4}{-\left(\sqrt{z^2-12}+2\right)}=\boxed{-2}$$

6. (2 pts – part (b) ONLY)

(a) In this case we can use the second formula because $x=0$ is completely inside this region.

$$\lim_{x\to 0} g(x)=\lim_{x\to 0}\ln(x+9)=\boxed{\ln(9)}$$

(b) Here we will need to look at the two one-sided limits because $x=-7$ is the "cut-off" point.

$$\lim_{x\to -7^-} g(x)=\lim_{x\to -7^-} x^2=49$$

$$\lim_{x\to -7^+} g(x)=\lim_{x\to -7^+}\ln(x+9)=\ln(2)$$

$$\lim_{x\to -7^-} g(x)\neq \lim_{x\to -7^+} g(x)$$

The two one-sided limits are not the same and so $\lim\limits_{x\to -7} g(x)$ doesn't exist.

9. (2 pts) In both of these limits the numerator is staying fixed at 10 and as *t* approaches -1 (from either side) we can see that 1+*t* is approaching zero. So, we have a fixed number divided by something increasingly smaller and so it should make some sense that both of these are going to either ∞ or $-\infty$ and this will depend upon the sign of the denominator

In the first case 1+*t* is negative since $t<-1$ and raising this to the 7th power will not change this. So, in this case we have a fixed positive number in the numerator divided by something increasingly smaller and negative and so the limit in this case will be $-\infty$.

In the second case 1+*t* is positive since $t > -1$ and raising this to the 7th power will not change this. So, in this case we have a fixed positive number in the numerator divided by an increasingly smaller and positive number and so the limit in this case will be ∞. Here are the official answers to this problem.

$$\lim_{t\to -1^-} \frac{10}{(1+t)^7} = -\infty \qquad \lim_{t\to -1^+} \frac{10}{(1+t)^7} = \infty$$

Not Graded

2.

(a)

$$\begin{aligned}\lim_{x\to 6}\left[h(x) - 3g(x) - 7f(x)\right] &= \lim_{x\to 6} h(x) - \lim_{x\to 6} 3g(x) - \lim_{x\to 6} 7f(x)\\ &= \lim_{x\to 6} h(x) - 3\lim_{x\to 6} g(x) - 7\lim_{x\to 6} f(x)\\ &= 1 - 3(9) - 7(-4) = \boxed{2}\end{aligned}$$

(b)

$$\begin{aligned}\lim_{x\to 6}\left[5 - f(x)\sqrt{g(x)}\right] &= \lim_{x\to 6} 5 - \lim_{x\to 6}\left[f(x)\sqrt{g(x)}\right]\\ &= \lim_{x\to 6} 5 - \left[\lim_{x\to 6} f(x)\right]\left[\lim_{x\to 6}\sqrt{g(x)}\right]\\ &= \lim_{x\to 6} 5 - \left[\lim_{x\to 6} f(x)\right]\left[\sqrt{\lim_{x\to 6} g(x)}\right]\\ &= 5 - (-4)\sqrt{9} = \boxed{17}\end{aligned}$$

(c)

$$\begin{aligned}\lim_{x\to 6}\frac{f(x) + h(x)}{g(x) - 7} &= \frac{\lim_{x\to 6}\left[f(x) + h(x)\right]}{\lim_{x\to 6}\left[g(x) - 7\right]}\\ &= \frac{\lim_{x\to 6} f(x) + \lim_{x\to 6} h(x)}{\lim_{x\to 6} g(x) - \lim_{x\to 6} 7}\\ &= \frac{-4+1}{9-7} = \boxed{-\frac{3}{2}}\end{aligned}$$

3. $\displaystyle \lim_{x\to 2}\frac{3x^2 - 7x + 2}{8 - 4x} = \lim_{x\to 2}\frac{(3x-1)(x-2)}{-4(x-2)} = \lim_{x\to 2}\frac{3x-1}{-4} = \boxed{-\frac{5}{4}}$

7. Note that we can't just cancel the *h*'s since once is inside the absolute value bars. So, we'll use the hint and recall that,

$$\left|h\right| = \begin{cases} h & \text{if } h \geq 0 \\ -h & \text{if } h < 0 \end{cases}$$

With this we can do the two one sided limits to eliminate the absolute value bars.

$$\lim_{h \to 0^-} \frac{\left|h\right|}{h} = \lim_{h \to 0^-} \frac{-h}{h} = \lim_{h \to 0^-} -1 = -1 \qquad \text{because } h < 0 \text{ in this case}$$

$$\lim_{h \to 0^+} \frac{\left|h\right|}{h} = \lim_{h \to 0^+} \frac{h}{h} = \lim_{h \to 0^-} 1 = 1 \qquad \text{because } h > 0 \text{ in this case}$$

The two one-sided limits are not the same and so $\lim\limits_{h \to 0} \frac{\left|h\right|}{h}$ does not exist.

8. In both of these limits the numerator is staying fixed at -3 and as *x* approaches 4 (from either side) we can see that *x* -4 is approaching zero. So, we have a fixed number divided by something increasingly smaller and so it should make some sense that both of these are going to either ∞ or $-\infty$ and this will depend upon the sign of the denominator

In the first case *x*-4 is negative since $x < 4$ and raising this to the 8th power will make it positive. So, in this case we have a fixed negative number in the numerator divided by something increasingly smaller and positive and so the limit in this case will be $-\infty$.

In the second case *x* -4 is positive since $x > 4$ and raising this to the 8th powers will not change this. So, in this case we have a fixed negative number in the numerator divided by an increasingly smaller and positive number and so the limit in this case will be $-\infty$.

Here are the official answers to this problem.

$$\lim_{x \to 4^-} \frac{-3}{\left(x-4\right)^8} = -\infty \qquad\qquad \lim_{x \to 4^+} \frac{-3}{\left(x-4\right)^8} = -\infty$$