2. (2 pts) Make sure you multiply out the denominator first before doing any actual limit work.

$$\lim_{x\to\infty}\frac{5x^3+1}{(2x+3)(1-x^2)}=\lim_{x\to\infty}\frac{5x^3+1}{2x-2x^3+3-3x^2}=\lim_{x\to\infty}\frac{x^3\left(5+\frac{1}{x^3}\right)}{x^3\left(\frac{2}{x^2}-2+\frac{3}{x^3}-\frac{3}{x}\right)}$$

$$=\lim_{x\to\infty}\frac{5+\frac{1}{x^3}}{\frac{2}{x^2}-2+\frac{3}{x^3}-\frac{3}{x}}=\boxed{-\frac{5}{2}}$$

4. (2 pts)

$$\lim_{w\to\infty}\frac{\sqrt{1+5w^2}}{3-w}=\lim_{w\to\infty}\frac{\sqrt{w^2\left(\frac{1}{w^2}+5\right)}}{w\left(\frac{3}{w}-1\right)}=\lim_{w\to\infty}\frac{|w|\sqrt{\frac{1}{w^2}+5}}{w\left(\frac{3}{w}-1\right)}\qquad\text{Assume } w>0 \text{ because } w\to\infty$$

$$=\lim_{w\to\infty}\frac{w\sqrt{\frac{1}{w^2}+5}}{w\left(\frac{3}{w}-1\right)}=\lim_{w\to\infty}\frac{\sqrt{\frac{1}{w^2}+5}}{\frac{3}{w}-1}=\frac{\sqrt{5}}{-1}=\boxed{-\sqrt{5}}$$

For the second limit the work will be identical until we get rid of the absolute value. In this case, we can assume that $w<0$ because $w\to-\infty$. So, picking up we get,

$$\lim_{w\to\infty}\frac{\sqrt{1+5w^2}}{3-w}\lim_{w\to\infty}\frac{|w|\sqrt{\frac{1}{w^2}+5}}{w\left(\frac{3}{w}-1\right)}=\lim_{w\to\infty}\frac{-w\sqrt{\frac{1}{w^2}+5}}{w\left(\frac{3}{w}-1\right)}=\lim_{w\to\infty}\frac{-\sqrt{\frac{1}{w^2}+5}}{\frac{3}{w}-1}=\frac{-\sqrt{5}}{-1}=\boxed{\sqrt{5}}$$

5. (2 pts) In this case we need to determine where the denominator will be zero. So, all we need to do is set the denominator equal to zero and solve.

$$x\cos(2x)+x=x\left(\cos(2x)+1\right)=0\quad\Rightarrow\quad x=0,\ \cos(2x)+1=0$$

So, we get $x=0$ as one point and we'll need to solve the second equation.

$$\cos(2x)=-1\quad\to\quad 2x=\pi+2\pi n\quad\to\quad x=\tfrac{1}{2}\pi+\pi n,\ n=0,\pm1,\pm2,\ldots$$

Note that in this case we can get the exact answer easily enough and from a unit circle we can see that there will be a single answer here. So, the function will not be continuous at,

$$x=0,\quad x=\tfrac{1}{2}\pi+\pi n,\ n=0,\pm1,\pm2,\ldots$$

9. (2 pts)

$$f'(x)=\lim_{h\to0}\frac{f(x+h)-f(x)}{h}=\lim_{h\to0}\frac{1+8(x+h)-2(x+h)^2-\left(1+8x-2x^2\right)}{h}$$

$$=\lim_{h\to0}\frac{1+8x+8h-2\left(x^2+2xh+h^2\right)-\left(1+8x-2x^2\right)}{h}$$

$$=\lim_{h\to0}\frac{8h-4xh-2h^2}{h}=\lim_{h\to0}(8-4x-2h)=8-4x\qquad\Rightarrow\qquad\boxed{f'(x)=8-4x}$$

13. (2 pts) To write down the equation of the tangent line we'll need,

$$g(17) = 191 \qquad g'(17) = 11$$

The tangent line is then,

$$y = 191 + 11(x - 17) = 11x + 4$$

So, the tangent line is identical to the original function. However, this should not surprise you as the original function was a line and so a tangent line to a line should be the same line!

Not Graded

1. $\lim\limits_{t \to -\infty} \dfrac{10 - t - 5t^3}{6t^2 - 7t} = \lim\limits_{t \to -\infty} \dfrac{t^2\left(\frac{10}{t^2} - \frac{1}{t} - 5t\right)}{t^2\left(6 - \frac{7}{t}\right)} = \lim\limits_{t \to -\infty} \dfrac{\frac{10}{t^2} - \frac{1}{t} - 5t}{6 - \frac{7}{t}} = \dfrac{\infty}{6} = \boxed{\infty}$

3. $\lim\limits_{z \to \infty} \dfrac{1 - z}{1 + z + z^2} = \lim\limits_{z \to \infty} \dfrac{z^2\left(\frac{1}{z^2} - \frac{1}{z}\right)}{z^2\left(\frac{1}{z^2} + \frac{1}{z} + 1\right)} = \lim\limits_{z \to \infty} \dfrac{\frac{1}{z^2} - \frac{1}{z}}{\frac{1}{z^2} + \frac{1}{z} + 1} = \dfrac{0}{1} = \boxed{0}$

6. Not much to do here. The function is continuous because it is a sum/product of continuous functions. Now,

$$f(5) = -140.899, \quad f(8) = 2948.071$$

So, we can see that $f(5) = -140.899 < 0 < 2948.071 = f(8)$ and so, by the IVT there is a number *c* such that $5 < c < 8$ such that $f(c) = 0$. Or in other words, *c* is a root of the function.

7. This problem seems quite complicated at first glance, but it isn't as bad as it might seem. We're being asked to find an interval with a width of no more than ½ in which the function will have a value of -1 in [-2,2]. Let's just need to start by evaluating the function at all the integers in this interval.

$$A(-2) = 0.149 \qquad A(-1) = 0 \qquad A(0) = -2.718 \qquad A(1) = 0 \qquad A(2) = 0.149$$

So, from these numbers the IVT tells us that the function will have a value of -1 in the intervals [-1,0] and [0,1]. These are both intervals of width 1 and so not quite what we want. However, with a couple of further computations we can get what we're looking for.

$$A(-0.5) = -1.588 \qquad A(0.5) = 1.588$$

The IVT now tells us that the function will have a value of -1 in the intervals $\left[-1, -\frac{1}{2}\right]$ and $\left[\frac{1}{2}, 1\right]$. Either of these will work as answers for this problem.

8.

$$g'(x) = \lim_{h\to 0}\frac{g(x+h)-g(x)}{h} = \lim_{h\to 0}\frac{4+11(x+h)-(4+11x)}{h} = \lim_{h\to 0}\frac{11h}{h} = \lim_{h\to 0}11 = 11$$

$$\boxed{g'(x) = 11}$$

10.

$$g'(w) = \lim_{h\to 0}\frac{g(w+h)-g(w)}{h} = \lim_{h\to 0}\frac{\left(\sqrt{4(w+h)+7}-\sqrt{4w+7}\right)}{h}\frac{\left(\sqrt{4(w+h)+7}+\sqrt{4w+7}\right)}{\left(\sqrt{4(w+h)+7}+\sqrt{4w+7}\right)}$$

$$= \lim_{h\to 0}\frac{(4w+4h+7)-(4w+7)}{h\left(\sqrt{4(w+h)+7}+\sqrt{4w+7}\right)} = \lim_{h\to 0}\frac{4h}{h\left(\sqrt{4(w+h)+7}+\sqrt{4w+7}\right)}$$

$$= \lim_{h\to 0}\frac{4}{\sqrt{4(w+h)+7}+\sqrt{4w+7}} = \frac{4}{2\sqrt{4w+7}} = \frac{2}{\sqrt{4w+7}} \quad \Rightarrow \quad \boxed{g'(w) = \frac{2}{\sqrt{4w+7}}}$$

11.

$$R'(t) = \lim_{h\to 0}\frac{R(t+h)-R(t)}{h} = \lim_{h\to 0}\frac{1}{h}\left(\frac{3}{(t+h)^2}-\frac{3}{t^2}\right) = \lim_{h\to 0}\frac{1}{h}\left(\frac{3t^2-3(t+h)^2}{t^2(t+h)^2}\right)$$

$$= \lim_{h\to 0}\frac{1}{h}\left(\frac{3t^2-3(t^2+2th+h^2)}{t^2(t+h)^2}\right) = \lim_{h\to 0}\frac{1}{h}\left(\frac{-6th-3h^2}{t^2(t+h)^2}\right) = \lim_{h\to 0}\frac{-6t-3h}{t^2(t+h)^2} = -\frac{6t}{t^4} = -\frac{6}{t^3}$$

$$\boxed{R'(t) = -\frac{6}{t^3}}$$

12. To answer this all we need is to evaluate the derivative we computed in **#10** and interpret the results.

$f'(0) = 8 > 0$	$f(x)$ is increasing at $x = 0$
$f'(6) = 8 - 4x = -16 < 0$	$f(x)$ is decreasing at $x = 6$

14. The function will stop changing if the derivative is ever zero. So we need to solve,

$$0 = R'(t) = -\frac{6}{t^3}$$

But this has no solutions and so the function will never stop changing.