

1. (2 pts) $h(t) = 10t^9 - 2t^{-\frac{3}{7}} + \frac{1}{9}t^{-2} - 12$

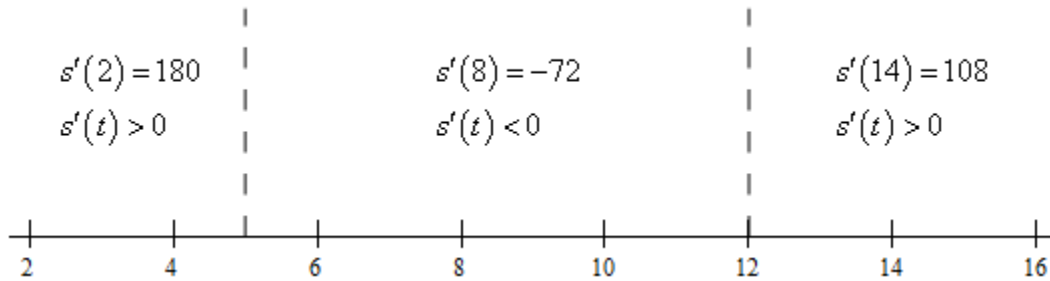
$$h'(t) = 90t^8 + \frac{6}{7}t^{-\frac{10}{7}} - \frac{2}{9}t^{-3}$$

4. (2 pts)

(a) $v(t) = s'(t) = 6t^2 - 102t + 360 = 6(t^2 - 17t + 60) = 6(t-5)(t-12)$

(b) The object will be at rest when $t = 5$ and $t = 12$.

(c) We need a number line here for this part and here is that.



So, it looks like,

$$\boxed{\text{Moving Right (Increasing) : } 0 \leq t < 5, \quad t > 12, \quad \text{Moving Left (Decreasing) : } 5 < t < 12}$$

7. (2 pts) $f'(x) = \frac{-6(10-x+3x^2) - (1-6x)(-1+6x)}{(10-x+3x^2)^2} = \frac{18x^2 - 6x - 59}{(10-x+3x^2)^2}$

11. (2 pts) $h'(\theta) = -3\sin^2(\theta) + 3\cos^2(\theta) - 4\theta^3 \sec \theta - \theta^4 \sec \theta \tan \theta$

13. (2 pts) First we'll need the derivative, set it equal to zero and solve.

$$P'(t) = 7 + 12\cos(t)$$

$$7 + 12\cos(t) = 0 \quad \rightarrow \quad \cos(t) = -\frac{7}{12} \quad \rightarrow \quad t = \cos^{-1}\left(-\frac{7}{12}\right) = 2.1936$$

From a quick sketch of a unit circle we can see that the second angle will be $2\pi - 2.1936 = 4.0896$. So, all possible solutions, and hence all places where the population isn't changing, are,

$$t = 2.1936 + 2\pi n \quad \text{OR} \quad t = 4.0896 + 2\pi n \quad n = 0, \pm 1, \pm 2, \dots$$

Now all that we need to do is plug in values of n and determine the solutions that are in the interval $[0, 15]$ (don't forget to change the months to years!).

$$\begin{aligned} n = 0: \quad t &= 2.1936 & \text{OR} & \quad t = 4.0896 \\ n = 1: \quad t &= 8.4768 & \text{OR} & \quad t = 10.3728 \\ n = 2: \quad t &= 14.7600 & \text{OR} & \quad \cancel{t = 16.6614} \end{aligned}$$

So, in the first 180 months (*i.e.* first 15 years) the population will not be changing at the five times listed above.

Not Graded

2. We need to find the derivative first, then the function and derivative evaluated at the point.

$$g(x) = 15x^5 - 2x \quad g'(x) = 75x^4 - 2 \quad g(-2) = -476 \quad g'(-2) = 1198$$

The tangent line is then,

$$y = -476 + 1198(x + 2) = 1198x + 1920$$

3. We'll need the derivative of both of these functions.

$$f'(x) = 3x^2 + 14x - 2 \quad g'(x) = -1 - 8x$$

Now, we know that if the tangent lines are going to be parallel then they must have the same slope and so the derivatives of these two functions must be equal at those points. So, all we need to do is set these equal and solve.

$$3x^2 + 14x - 2 = -1 - 8x$$

$$3x^2 + 22x - 1 = 0 \quad \Rightarrow \quad x = \frac{-22 \pm \sqrt{22^2 - 4(3)(-1)}}{6} = \frac{-22 \pm \sqrt{496}}{6} = \frac{-11 \pm 2\sqrt{31}}{3} = -7.3785, 0.0452$$

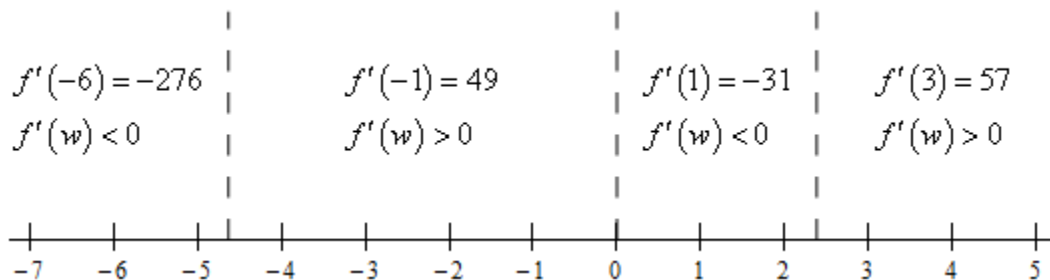
So, the tangent lines will be parallel at -7.3785 and 0.0452.

5. We'll need the derivative and where it is zero.

$$f'(w) = 4w^3 + 9w^2 - 44w = w(4w^2 + 9w - 44)$$

$$f'(w) = 0 \quad \rightarrow \quad w = 0, w = \frac{-9 \pm \sqrt{81 - 4(4)(-44)}}{8} = \frac{-9 \pm \sqrt{785}}{8} = -4.6272, 2.3772$$

Do, not worry about the "messy" numbers here. They will happen on occasion so you need to be able to deal with them. Here is a number line for this problem.



So, in the interval $[8,4]$ we have,

$$\text{Increasing : } -4.6272 < w < 0, 2.3773 < w \leq 4$$

$$\text{Decreasing : } -8 \leq w < -4.6272, 0 < w < 2.3773$$

Therefore we are decreasing

$$\frac{3.3728 + 2.3773}{12} = 0.479175 \quad \Rightarrow \quad \boxed{47.92\%}$$

The function is decreasing 47.92% in the range $[-8,4]$.

$$6. \quad R'(z) = z^{-\frac{1}{2}} \left(z^{\frac{4}{3}} - z^{\frac{1}{5}} \right) + \left(2z^{\frac{1}{2}} + 3 \right) \left(\frac{4}{3} z^{\frac{1}{3}} - \frac{1}{5} z^{-\frac{4}{5}} \right)$$

8. We'll need the derivative of the function to answer this so,

$$V'(t) = \frac{2t(2t^2 - 3t + 4) - t^2(4t - 3)}{(2t^2 - 3t + 4)^2} = \frac{8t - 3t^2}{(2t^2 - 3t + 4)^2}$$

Now, the function will not be changing where the derivative is zero. In this case, because we're working with a rational expression this will only happen if the numerator is zero (and the denominator is not simultaneously zero). Setting the numerator equal to zero and solving gives,

$$8t - 3t^2 = t(8 - 3t) = 0 \quad \rightarrow \quad t = 0, \frac{8}{3}$$

A quick check shows that the denominator is not also zero at these two points and so these are the two values for which the function is not changing.

$$9. \quad g'(t) = 4 \sec(t) \tan(t) + 8 \csc(t) \cot(t) + \sin(t) + t \cos(t)$$

$$10. \quad \frac{dy}{dx} = \frac{\sec^2(x)(3 - \cot(x)) - \csc^2(x)(5 + \tan(x))}{(3 - \cot(x))^2}$$

12. We need to find the derivative first, then the function and derivative evaluated at the point.

$$\frac{dy}{dx} = \frac{(0)(1 - \cos(x)) - 3 \sin x}{(1 - \cos(x))^2} = \frac{-3 \sin x}{(1 - \cos(x))^2} \quad y|_{x=\pi} = \frac{3}{2} \quad \frac{dy}{dx}|_{x=\pi} = 0$$

So, the tangent line is,

$$y = \frac{3}{2} + 0(x - \pi) \quad \Rightarrow \quad y = \frac{3}{2}$$