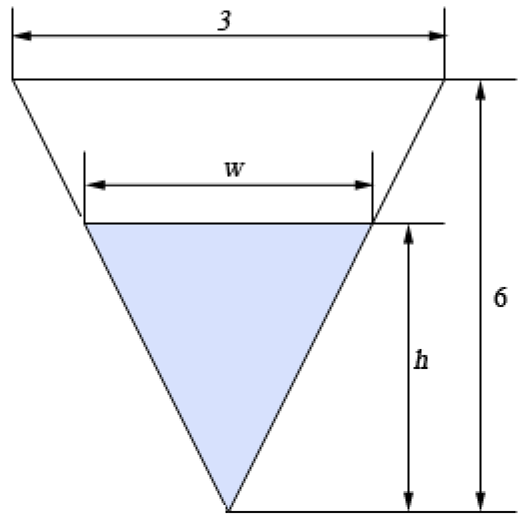
1. (2 pts)

$$3y^2y'\mathbf{e}^{x^2} + 2xy^3\mathbf{e}^{x^2} + 4x^3 = -\cos(y)\,y'$$

$$\left(3y^2\mathbf{e}^{x^2} + \cos(y)\right)y' = -2xy^3\mathbf{e}^{x^2} - 4x^3 \qquad \Rightarrow \qquad \boxed{y' = \frac{-2xy^3\mathbf{e}^{x^2} - 4x^3}{3y^2\mathbf{e}^{x^2} + \cos(y)}}$$

5. (2 pts) The important sketch for this part is the sketch of the end of the tank so here is that,



Here *h* represents the height of the water and *w* represents the maximum width of the water. Now, the volume of water in the tank is the area of the base times the length of the tank or,

$$V = \tfrac{1}{2}(base)(height)(length) = \tfrac{1}{2}(w)(h)(10) = 5wh$$

We'll need to eliminate one of the variables and because we're asked to find the rate at which the water is changing, *i.e.* h' we'll eliminate *w*. Using similar triangles we see that,

$$\frac{w}{h} = \frac{3}{6} \qquad \rightarrow \qquad w = \frac{1}{2}h$$

The volume formula is then : $V = \frac{5}{2}h^2 \qquad \Rightarrow \qquad V' = 5hh'$

We know that $V' = -\frac{1}{2}$ and again using the similar triangles we see that when $w = \frac{18}{12} = \frac{3}{2}$ we get,

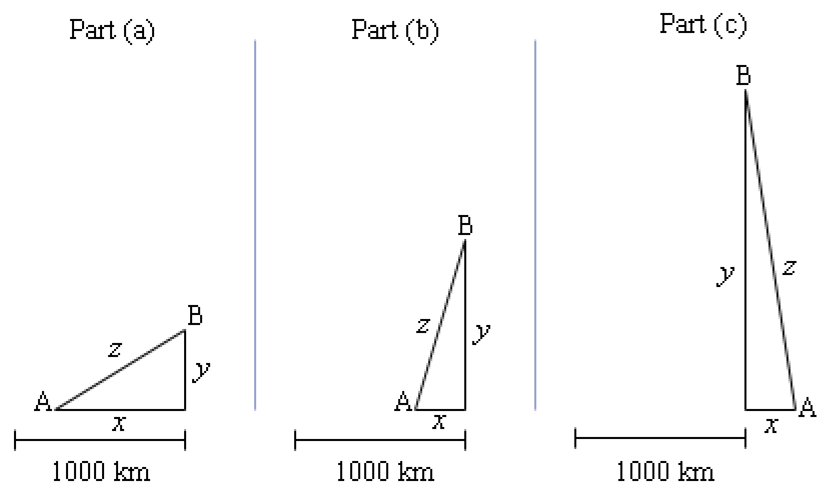
$$\frac{3}{2} = \frac{1}{2}h \qquad \Rightarrow \qquad h = 3$$

So, plugging in and solving gives,

$$-\tfrac{1}{2} = 5(3)h' \qquad \Rightarrow \qquad h' = -\tfrac{1}{30}$$

The height is then decreasing at a rate of $\frac{1}{30}$ ft/min.

6. (2 pts) Here is the sketch for each part of this problem and notice that for **(c)** we've actually moved past the starting point of boat B.



In each case we're going to need to find z' and to do this we'll use the Pythagorean Theorem and so we may as well set that up now and then we'll actually work the problem.

$$x^2 + y^2 = z^2 \qquad \Rightarrow \qquad z' = \tfrac{1}{z}\left(x\,x' + y\,y'\right)$$

(a) Here's all the important quantities for this part.

$$x = 1000 - 100(3) = 700 \qquad x' = -100 \qquad y = 65(3) = 195 \qquad y' = 65$$

$$z = \sqrt{700^2 + 195^2} = 726.6533$$

The rate at which the distance between the two boats is changing is,

$$z' = \tfrac{1}{726.6533}\left((700)(-100) + (195)(65)\right) = \underline{-78.8891 \text{ km/hr}}$$

So, in this case the distance is **decreasing**.

(b) Here's all the important quantities for this part.

$$x = 1000 - 100(8) = 200 \qquad x' = -100 \qquad y = 65(8) = 520 \qquad y' = 65$$

$$z = \sqrt{200^2 + 520^2} = 557.1355$$

The rate at which the distance between the two boats is changing is,

$$z' = \tfrac{1}{557.1355}\left((200)(-100) + (520)(65)\right) = \underline{24.7696 \text{ km/hr}}$$

So, in this case the distance is **increasing**.

(c) Here's all the important quantities for this part.

$$x = 100(12) - 1000 = 200 \qquad x' = 100 \qquad y = 65(12) = 780 \qquad y' = 65$$

$$z = \sqrt{200^2 + 780^2} = 802.2329$$

The rate at which the distance between the two boats is changing is,

$$z' = \tfrac{1}{802.2329}\left((200)(100) + (780)(65)\right) = \underline{88.1290 \text{ mph}}$$

So, in this case the distance is **increasing**.

8. (2 pts) Remember sometimes the product and/or quotient rule will be needed for the higher order derivatives even if they weren't needed for the first derivative.

$$y' = 2x\cos\left(1+x^2\right) - \frac{2x}{1+x^2}$$

$$y'' = 2\cos\left(1+x^2\right) - 4x^2\sin\left(1+x^2\right) - \frac{2\left(1+x^2\right) - 2x(2x)}{\left(1+x^2\right)^2}$$

$$= \boxed{2\cos\left(1+x^2\right) - 4x^2\sin\left(1+x^2\right) - \frac{2-2x^2}{\left(1+x^2\right)^2}}$$

10. (2 pts)

$$g'(x) = -4\sin(4x) + \frac{6x^2}{2x^3} - 7\mathbf{e}^{7x} = -4\sin(4x) + \frac{3}{x} - 7\mathbf{e}^{7x}$$

$$g^{(2)}(x) = g''(x) = -16\cos(4x) - \frac{3}{x^2} - 49\mathbf{e}^{7x}$$

$$\boxed{g^{(3)}(x) = g'''(x) = 64\sin(4x) + \frac{6}{x^3} - 343\mathbf{e}^{7x}}$$

Not Graded

2.

$$\frac{2xy^4 + 4x^2y^3y'}{x^2y^4} = 2yy'$$

$$\frac{2}{x} + \frac{4y'}{y} = 2yy'$$

$$\frac{2}{x} = \left(2y - \frac{4}{y}\right)y' \qquad \Rightarrow \qquad \boxed{y' = \frac{2}{x\left(2y - 4y^{-1}\right)}}$$

3. We'll first need to do some implicit differentiation to find the derivative.

$$-\sin\left(x-y^2\right)\left(1-2yy'\right) = -y - xy'$$

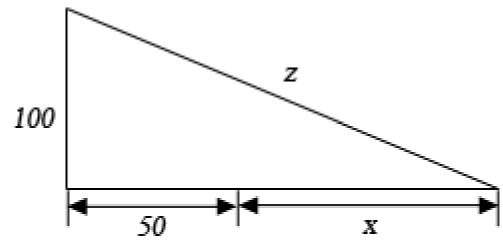
$$2yy'\sin\left(x-y^2\right) - \sin\left(x-y^2\right) = -y - xy'$$

$$\left(2y\sin\left(x-y^2\right)+x\right)y' = -y + \sin\left(x-y^2\right) \quad \Rightarrow \quad y' = \frac{-y+\sin\left(x-y^2\right)}{2y\sin\left(x-y^2\right)+x}$$

Now evaluate the derivative at the point and then write down the equation of the tangent line.

$$y'\big|_{x=4,y=2} = -\tfrac{1}{2} \qquad\qquad \boxed{y = 2 - \tfrac{1}{2}(x-4) = -\tfrac{1}{2}x + 4}$$

4. First, let's start off with a quick sketch of the situation that will work for both cases.



Here's the equation that will govern both parts.

$$100^2+\left(50+x\right)^2=z^2 \qquad \Rightarrow \qquad 2\left(50+x\right)x'=2zz'$$

Now the work for each part.

(a) Here we have,

$$x=\left(1.5\right)\left(30\right)=45\ ft \qquad x'=1.5 \qquad z=\sqrt{100^2+\left(50+45\right)^2}=\sqrt{19025}=137.931$$

In this case we are looking for z' so solving the equation above for z' plugging in known quantities and evaluating gives,

$$z'=\frac{\left(50+x\right)x'}{z}=\frac{\left(50+45\right)\left(1.5\right)}{137.931}=1.0331$$

So, the string is increasing in length at a rate of 1.0331 ft/sec.

(b) In this case we have to be a little careful. First the initial value of *z* is,

$$z=\sqrt{100^2+50^2}=\sqrt{12500}=111.8034$$

Now, we are told that the string is increasing at 1.5 ft/sec so after 30 seconds we will have

$$z=111.8034+\left(1.5\right)\left(30\right)=156.8034$$

i.e. the initial value of *z* plus the amount string let out in 30 seconds. This means that we'll have the following quantities for this part,

$$50+x=\sqrt{156.8034^2-100^2}=120.7779 \qquad x=70.7779 \qquad z'=1.5$$

In this case we are after x' so solve the equation for x', plug in and evaluate.

$$x'=\frac{zz'}{50+x}=\frac{\left(156.8034\right)\left(1.5\right)}{120.7779}=1.9474$$

So the person is moving away at a rate of 1.9474 ft/sec.

7.

$$f\left(x\right)=5x^{-4}-x^{\frac{3}{2}}+\tfrac{1}{6}x^{-1} \qquad f'\left(x\right)=-20x^{-5}-\tfrac{3}{2}x^{\frac{1}{2}}-\tfrac{1}{6}x^{-2} \qquad \boxed{f''\left(x\right)=100x^{-6}-\tfrac{3}{4}x^{-\frac{1}{2}}+\tfrac{1}{3}x^{-3}}$$

9.

$$P'\left(z\right)=4\sec^2\left(1+4z\right)$$

$$P'\left(z\right)=8\sec\left(1+4z\right)\left[\sec\left(1+4z\right)\tan\left(1+4z\right)\left(4\right)\right]=\boxed{32\sec^2\left(1+4z\right)\tan\left(1+4z\right)}$$