Math 2413 **Homework Set 7 – Solutions** **10 Points**

1. (2 pts) First the derivative.

$$g'(t) = 2(t-1)\left(t^2-16\right)^3 + 3(t-1)^2\left(t^2-16\right)^2(2t)$$
$$= 2(t-1)\left(t^2-16\right)^2\left[\left(t^2-16\right)+3t(t-1)\right] = 2(t-1)\left(t^2-16\right)^2\left(4t^2-3t-16\right)$$

This exists everywhere and so the critical points will only come from where the derivative is zero.

$$t-1=0 \qquad \to \qquad \boxed{t=1}$$
$$t^2-16=0 \qquad \to \qquad \boxed{t=\pm 4}$$
$$4t^2-3t-16=0 \qquad \to \qquad \boxed{t=\tfrac{3\pm\sqrt{265}}{8}=-1.6599,\ 2.4099}$$

So, we have the five critical points shown above.

5. (2 pts) First the derivative and because the derivative exists everywhere the critical points will only be where the derivative is zero.

$$R(t) = 4\mathbf{e}^{1-(t-3)^2} + (4t-10)\left[-2(t-3)\right]\mathbf{e}^{1-(t-3)^2} = \left(-8t^2+44t-56\right)\mathbf{e}^{1-(t-3)^2}$$
$$= -4(2t-7)(t-2)\mathbf{e}^{1-(t-3)^2}$$

Because we know that exponential functions are not zero we can see that the only two critical points are $t=2$ and $t=\frac{7}{2}$.

8. (2 pts) We need the critical points that are in the interval. We found all of the critical points in **#3.** Here are the critical points that fall in the interval.

$$x=0 \qquad\qquad x=3$$

Now all we need to do is evaluate the function at these points and the endpoints. Here are the function evaluations.

$$g(-1)=-69 \qquad g(0)=12 \qquad g(3)=795 \qquad g(4)=-244$$

The absolute maximum is then 795 at $x=3$ and the absolute minimum is -244 at $x=4$.

11. (2 pts) All that we're asking here is to find the absolute maximum of the function in the interval [0,3] and we know how to do that. We found the critical points of this function in **#5** and only $t=2$ is in the interval. So, all we need to do is check the function at this point and the endpoints.

$$R(0)=3.9966 \qquad R(2)=2 \qquad R(3)=9.4366$$

The absolute maximum is 9.4366 and the absolute minimum 2 is and so the chemical process is safe in the first three hours.

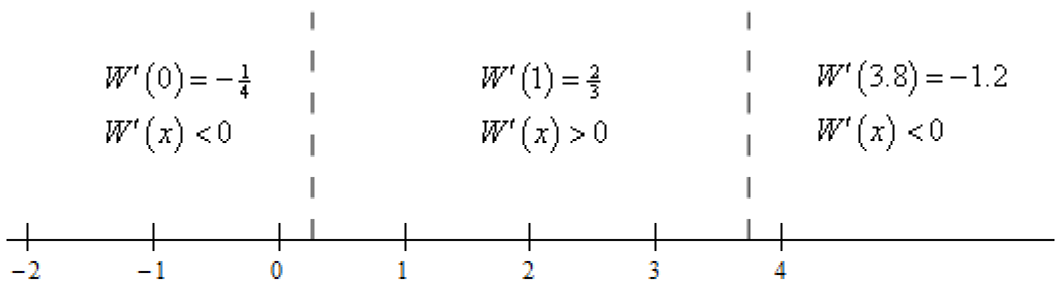
13. (2 pts) In this case, unlike all the previous problems, we've not found the critical points ahead of time so first let's find the derivative.

$$W'(x) = x - \frac{2}{8-2x} = \frac{x(8-2x)-2}{8-2x} = \frac{-2x^2+8x-2}{8-2x} = \frac{x^2-4x+1}{x-4}$$

Okay, we can see that the derivative doesn't exist at $x = 4$, but the function also doesn't exist there so this is not a critical point. The derivative will be zero where the numerator is zero so solving that gives,

$$x^2 - 4x + 1 = 0 \qquad x = \frac{4 \pm \sqrt{16-4(1)(1)}}{2} = \frac{4 \pm \sqrt{12}}{2} = 2 \pm \sqrt{3} = 0.2679,\ 3.7321$$

Now, we need the number line for this problem.



So, on interval given the increasing/decreasing information is,

$$\text{Increasing}: \ 0.2679 < x < 3.7321 \qquad \text{Decreasing}: \ -5 \le x < 0.2679,\ 3.7321 < x < 4$$

We can also see then that we have the following classifications of the critical points.

$$x = 0.2679: \text{Relative Minimum} \qquad x = 3.7321: \text{Relative Maximum}$$

Not Graded

2. First the derivative and because the derivative exists everywhere the critical points will only be where the derivative is zero.

$$P'(t) = 8\cos(4t) + 1 \qquad \rightarrow \qquad \cos(4t) = -\tfrac{1}{8} \qquad \rightarrow \qquad 4t = \cos^{-1}\left(-\tfrac{1}{8}\right) = 1.6961$$

From a unit circle we can see that the second angle will be $2\pi - 1.6961 = 4.5871$. So, all the critical points are,

$$\begin{aligned} 4t &= 1.6961 + 2\pi n \\ 4t &= 4.5871 + 2\pi n \end{aligned} \qquad \Rightarrow \qquad \boxed{\begin{aligned} t &= 0.4240 + \tfrac{\pi n}{2} \\ t &= 1.1468 + \tfrac{\pi n}{2} \end{aligned} \qquad n = 0, \pm 1, \pm 2, \ldots}$$

3. First the derivative and because the derivative exists everywhere the critical points will only be where the derivative is zero.

$$g'(x) = 240x^2 - 20x^3 - 20x^4 = -20x^2\left(x^2 + x - 12\right) = -20x^2(x-3)(x+4)$$

So, the three critical points are : $x = -4$, $x = 0$ and $x = 3$.

4. First the derivative.

$$V'(t) = 2t(5-t)^{\frac{1}{3}} - \tfrac{1}{3}t^2(5-t)^{-\frac{2}{3}} = 2t(5-t)^{\frac{1}{3}} - \frac{t^2}{3(5-t)^{\frac{2}{3}}} = \frac{6t(5-t)-t^2}{3(5-t)^{\frac{2}{3}}} = \frac{30t-7t^2}{3(5-t)^{\frac{2}{3}}}$$

So, from the numerator we can see that if $t=0$ and/or $t=\frac{30}{7}$ the derivative will be zero and so are critical points. Also, from the denominator if $t=5$ the derivative will not exist (and the function does exist here) and so will be a critical point. These are the three critical points.

6. Rel. Max : *b*, *d*, *f* Rel. Min. : *c*, *e* Abs. Max. : *d* Abs. Min. : *g*

7. We need the critical points that are in the interval. We found all of the critical points in **#3.** and all of them are in the interval so here they are critical points for this function.

$$t=1, \quad t=\pm 4, \quad t=-1.6599,\ 2.4099$$

Now all we need to do is evaluate the function at these points and the endpoints. Here are the function evaluations.

$$g(-5)=26244 \qquad g(-4)=0 \qquad g(-1.6599)=-16438.4227$$
$$g(1)=0 \qquad g(2.4099)=-2104.7654 \qquad g(4)=0 \qquad g(5)=11664$$

The absolute maximum is then 26244 at $t=-5$ and the absolute minimum is -16438.4227 at $t=-1.6599$.

9. All that we're asking here is to find the absolute maximum of the function in the interval [0,5] and we know how to do that. We found the critical points of this function in **#4** and both are in the interval so here is the function evaluated at these two critical points and at the endpoints.

$$V(0)=20 \qquad V\left(\tfrac{30}{7}\right)=36.4186 \qquad V(5)=20$$

Note that one of the endpoints was also a critical point and there is nothing wrong with this. So we can see from this that the absolute maximum is 36.4186 and so the battery will not be safe in the first 5 hours of operation.

10. All that we're asking here is to find the absolute maximum of the function in the interval [0,2.5] and we know how to do that. We found the critical points of this function in **#2** and by plugging in values of *n* we can find the critical points in the interval. Here are those critical points.

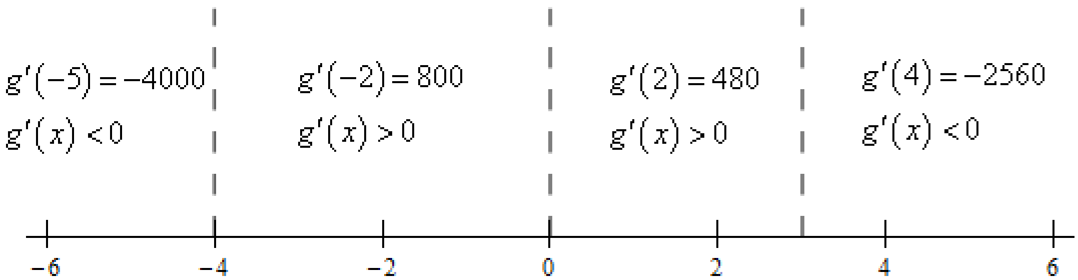
$$t=0.4240 \qquad t=1.1468 \qquad t=1.9948$$

Here is the function evaluated at the critical points and the endpoints.

$$P(0)=15 \qquad P(0.4240)=17.4083 \qquad P(1.1468)=14.1625$$
$$P(1.9948)=18.9791 \qquad P(2.5)=16.4119$$

The absolute maximum population is then 1897.91 at $t = 1.9948$ and the absolute minimum population is 1416.25 at $t = 1.1468$.

12. We found the critical points in **#3** ($x = -4$, $x = 0$ and $x = 3$) so all we need to do is find the increasing/decreasing information. Here is the number line for this problem.



The increasing/decreasing information is then,

$$\text{Increasing}: \quad -4 < x < 0, \quad 0 < x < 3 \qquad\qquad \text{Decreasing}: \quad -\infty < x < -4, \; 3 < x < \infty$$

We can also see then that we have the following classifications of the critical points.

$$x = -4: \text{Relative Minimum} \qquad x = 0: \text{Neither} \qquad x = 3: \text{Relative Maximum}$$