The Shape of a Graph, Part II

1. Determine the intervals where $R(x) = 10 + 8x - \frac{28}{5}x^6 - \frac{1}{3}x^7 + \frac{1}{8}x^8$ is concave up and concave down. What are the inflection points for this function?

2. Use $g(x) = 12 + 80x^3 - 5x^4 - 4x^5$ for this problem.

(a) What are the intervals of increase/decrease for this function?
(b) What are the relative extrema of this function?
(c) Find the intervals of concave up/concave down for this function.
(d) Find all the inflection points for this function.
(e) Use the information above to sketch the graph of this function.

3. Suppose that you know that the critical points of $f(x)$ are $x = -10$, $x = 2$, and $x = 6$ and that the second derivative of $f''(x) = 4x^3 + 36x^2 - 96x - 560$. If possible, classify the critical points. If it is not possible clearly explain why it is not possible.

The Mean Value Theorem

4. Verify that $g(x) = x^3 + 4x^2 - 2x - 9$ satisfies the conditions of the Mean Value Theorem on [1, 3] and find all values of *c* that satisfy the conclusion of the Mean Value Theorem on [1, 3].

5. Suppose that we know that $f(x)$ is a continuous and differentiable function and that $f(-2) = f(1) = 569$. Show that $f(x)$ must have a critical point in the interval [-2, 1].

Optimization

In order to receive any credit for problems 6 – 8 you MUST use Calculus techniques to find the answer. Any decimal work should include at least 4 decimal places.

6. We want to construct a box whose base width is 2 times its base length and must have a volume of 100m^3. If the sides cost \$5/m, the top costs \$10/m and the bottom costs \$20/m determine the dimensions of the box that will minimize the cost of construction.

7. Find the point(s) on $x = y^2 - 2$ that are closest to the point (1, 0).

8. We have a piece of wire that is 20 cm long and we're going to cut it into two pieces. One piece will be bent into a square and the other will be bent into a circle. Determine where the wire should be cut so that the enclosed areas will a maximum. Note that it is possible to have the whole piece of wire go to the square or to the circle so you'll need to take that into account as well.

Continued on Back $\Rightarrow$

<u>L'Hospitals Rule</u>

Evaluate each of the following limits.

9. $\lim\limits_{t \to \infty} \dfrac{3t - 7t^2}{12t^2 + 5t - 10}$

10. $\lim\limits_{x \to -1} \dfrac{(x+1)^2}{\mathbf{e}^{2x+2} + 3x^2 + 4x}$

11. $\lim\limits_{z \to \infty} \dfrac{6z + \mathbf{e}^{7z}}{5z + 2\mathbf{e}^{3z}}$

12. $\lim\limits_{x \to \infty} \left[x \sin\left(\dfrac{6}{x} \right) \right]$

<u>Differentials</u>

Compute the differential for each of the following functions.

13. $u = \tan\left(5t^2 - t\right)$

14. $f(x) = x^2 \mathbf{e}^{9-x}$