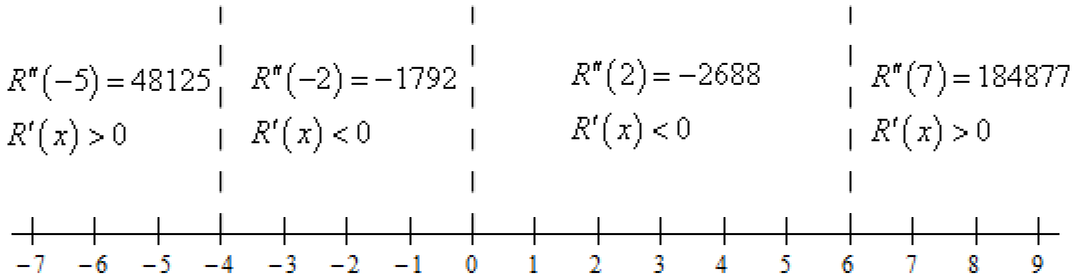
1. (2 pts) First we need the 1st and 2nd derivatives of this function.

$$R'(x) = 8 - \tfrac{168}{5}x^5 - \tfrac{7}{3}x^6 + x^7 \qquad R''(x) = -168x^4 - 14x^5 + 7x^6 = 7x^4(x-6)(x+4)$$

So, the possible inflection points are then : $x = -4$, $x = 0$ and $x = 6$. Here is a number line.



So, the function is concave up/down information is then,

$$\text{Concave Up : } -\infty < x < -4,\ 6 < x < \infty \qquad \text{Concave Down : } -4 < x < 0, 0 < x < 6$$

This also means that the inflections points are $x = -4$ and $x = 6$.

4. (2 pts) This is a polynomial and so is continuous and differentiable everywhere, in particular, on the given interval and so the Mean Value Theorem can be used.

$$f'(x) = 3x^2 + 8x - 2 \qquad f(1) = -6 \qquad f(3) = 48$$

Applying the Mean Value Theorem and solving gives,

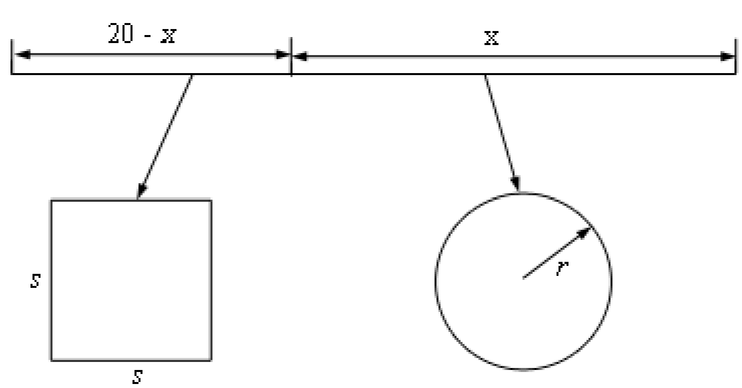
$$3c^2 + 8c - 2 = f'(c) = \frac{f(3) - f(1)}{3-1} = \frac{54}{2} = 27$$

$$3c^2 + 8c - 2 = 27$$

$$3c^2 + 8c - 29 = 0 \qquad \Rightarrow \qquad c = \tfrac{-8 \pm \sqrt{64 - 4(3)(-29)}}{6} = \tfrac{-8 \pm \sqrt{412}}{6} = -4.7163,\ 2.0496$$

The first solution is not in the interval while the second solution is so the only value that satisfies the Mean Value Theorem is 2.0496.

8. (2 pts) Here's a quick sketch of the situation.



Okay, here is what we know about these two objects and their perimeters compared to how much wire we used to make each object.

$$\text{Perimeter of Square : } 4s = 20 - x \qquad \Rightarrow \qquad s = \tfrac{1}{4}(20 - x)$$

$$\text{Circumference of circle : } 2\pi r = x \qquad \Rightarrow \qquad r = \frac{x}{2\pi}$$

$$\text{Total Length : } 20 = 4s + 2\pi r$$

There are two ways to proceed with solving this problem. One is to do the traditional optimization with constraint. Here are the equations that we'll need to deal with to do that.

$$\text{Maximize : } A = s^2 + \pi r^2$$

$$\text{Constraint : } 20 = 4s + 2\pi r$$

In this case we would solve the constraint for *s* or *r*, plug into the equation and maximize. Get *s* or *r* then determine what *x* needs to be.

The other way is to use the fact that the constraint is built into the perimeter and circumference formulas above and solve them in terms of *x* as I've done and plug these into the area and solve directly for *x*. In this case the area becomes

$$A(x) = s^2 + \pi r^2 = \frac{1}{16}(20 - x)^2 + \pi\left(\frac{x}{2\pi}\right)^2 = \frac{1}{16}(20 - x)^2 + \frac{x^2}{4\pi}$$

Now, if we just differentiate a couple of times we get,

$$A'(x) = -\frac{1}{8}(20 - x) + \frac{x}{2\pi} = \left(\frac{1}{2\pi} + \frac{1}{8}\right)x - \frac{5}{2} \qquad\qquad A''(x) = \frac{1}{8} + \frac{1}{2\pi}$$

The only critical point in this case is

$$x = \frac{5/2}{\frac{1}{8} + \frac{1}{2\pi}} = 8.7980$$

Now, we've got a small problem here. The second derivative is always positive so that means that this critical point is in fact a relative minimum. In other words, if we use 8.7980 cm's for the circle and 11.2020 cm's for the square we will get the minimum enclosed area. This was not what the problem asked for however. So, it looks like what we need is to consider using all the wire for the square or all the wire for the circle. So, here are the areas if we use all the wire for the square and all the wire for the circle.

$$\text{Square : } A = \left(\frac{20}{4}\right)^2 = 25$$

$$\text{Circle : } A = \pi\left(\frac{20}{2\pi}\right)^2 = 31.8310$$

Don't forget that you'll need to determine the length of the side (wire length divided by 4 to get equal sides) and radius (30 = Circumference) before doing these computations! Note that if we computed the area at the critical point from above we would get $A(8.7980) = 14.0025$ which is smaller than both the critical points and from our work with Absolute Extrema we know that this would give the absolute minimum area enclosed.

So, it looks like we'll need to use all the wire for the circle if we want to maximize the area.

9. (2 pts) $\lim\limits_{t\to\infty}\frac{3t-7t^2}{12t^2+5t-10}=\lim\limits_{t\to\infty}\frac{3-14t}{24t+5}=\lim\limits_{t\to\infty}\frac{-14}{24}=\frac{-14}{24}=\boxed{-\frac{7}{12}}$

11. (2 pts) $\lim\limits_{z\to\infty}\frac{6z+\mathbf{e}^{7z}}{5z+2\mathbf{e}^{3z}}=\lim\limits_{z\to\infty}\frac{6+7\mathbf{e}^{7z}}{5+6\mathbf{e}^{3z}}=\lim\limits_{z\to\infty}\frac{49\mathbf{e}^{7z}}{18\mathbf{e}^{3z}}=\lim\limits_{z\to\infty}\frac{49}{18}\mathbf{e}^{4z}=\infty$

Not Graded

2. We looked at this problem in **#3** and **#12** of the previous homework assignment. So, the critical points are : $x=-4$, $x=0$ and $x=3$ and the increasing/decreasing information is,

$$\text{Increasing}: \ -4<x<0,\ \ 0<x<3 \qquad\qquad \text{Decreasing}: \ -\infty<x<-4,\ 3<x<\infty$$

We also found the following relative extrema information in **#12**. Here is that information.

$$x=-4: \text{Relative Minimum} \qquad x=0: \text{Neither} \qquad x=3: \text{Relative Maximum}$$

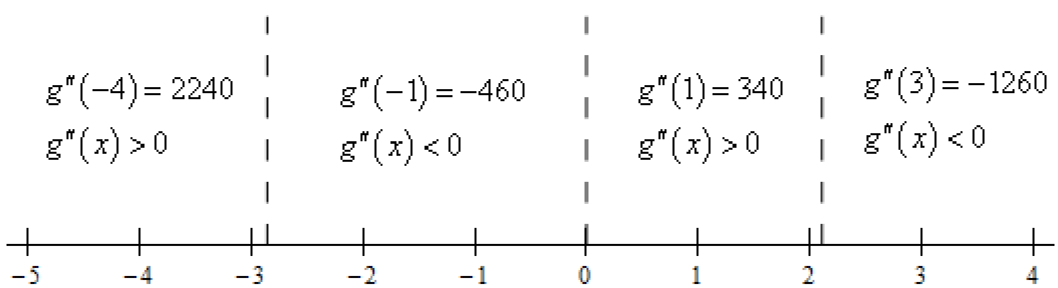
We can now pickup the rest of the problem. Here is the second derivative for this function.

$$g''(x)=480x-60x^2-80x^3=-20x\left(4x^2+3x-24\right)$$

The possible inflection points are then,

$$x=0, \quad x=\frac{-3\pm\sqrt{9-4(4)(-24)}}{8}=\frac{-3\pm\sqrt{393}}{8}=-2.8530,\ 2.1030$$

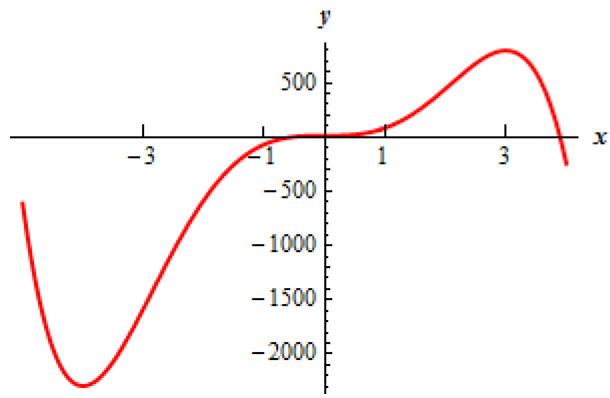
Here is a number line for the second derivative.



So, the function is concave up/down information is then,

$$\text{Concave Up}: \ -\infty<x<-2.8530,\ 0<x<2.1030$$
$$\text{Concave Down}: \ -2.8230<x<0,\ \ 2.1030<x<\infty$$

This also means that the inflections points are $x=-4$, $x=0$ and $x=3$. Finally here is a quick sketch.



3. All we can do here is try the second derivative test.

$f''(-10)=0$ Don't Know - Test says nothing in this case.

$f''(2)=-576$ Relative Maximum

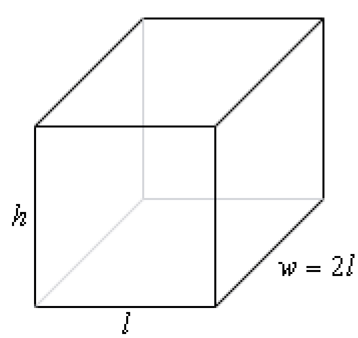
$f''(6)=1024$ Relative Minimum

5. This is not as tricky as it might first appear to be. Because we're told that the function is continuous and differentiable then we know that we can apply the Mean Value Theorem. So let's do that and see what we get.

$$f'(c)=\frac{f(1)-f(-2)}{1-(-2)}=\frac{569-569}{3}=0$$

So, the Mean Value Theorem tells us that somewhere in the interval [-2, 1] is a number, *c*, such that $f'(c)=0$. But that means *c* is a critical point of the function and it is in the interval and so we're done.

6. Here is a sketch of the box.



Here's the equations we'll need for this problem.

$$\text{Minimize}: C=5(2hl+2hw)+10(lw)+20(lw)=30hl+60l^2$$

$$\text{Contraint}:\ 100=hwl=2hl^2$$

Solving the constraint for *h* and plugging into the cost gives,

$$h=\tfrac{50}{l^2} \qquad \rightarrow \qquad C(l)=30\left(\tfrac{50}{l^2}\right)l+60l^2=\tfrac{1500}{l}+60l^2=\tfrac{1500+60l^3}{l}$$

Now we'll need a couple of derivatives.

$$C'(l) = -\frac{1500}{l^2} + 120l = \frac{120l^3 - 1500}{l^2} \qquad C''(l) = \frac{3000}{l^3} + 120$$

So, it looks like we have two critical points,

$$l = 0 \qquad l = \sqrt[3]{\frac{1500}{120}} = \sqrt[3]{\frac{25}{2}} = 2.3208$$

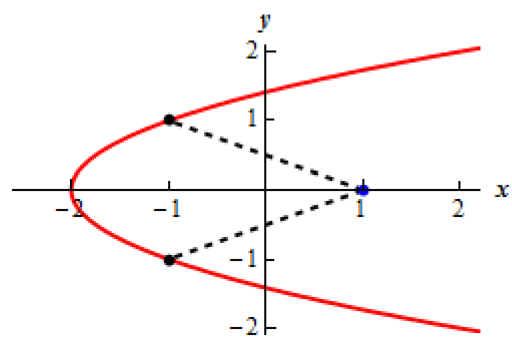
Because we have a box the first doesn't make any sense and as long as *l* is positive we can see that the second derivative will be positive and so this must give a minimum value.

The dimensions are then,

$$l = 2.3208 \qquad w = 2(2.3208) = 4.6416 \qquad h = \frac{50}{2.3208^2} = 9.2831$$

#7. In this case we want to minimize the square of the distance between (1, 0) and (*x*, *y*), a point on the graph and the equation of the graph is the constraint.

$$d^2 = (x-1)^2 + y^2 \qquad x = y^2 - 2$$



Now, solve the graph equation for y^2 and plug this into the square of the distance to get the function we'll differentiate.

$$f(x) = x^2 - 2x + 1 + (x+2) = x^2 - x + 3 \qquad f'(x) = 2x - 1 \qquad f''(x) = 2$$

The only critical point we have is $x = \frac{1}{2}$ and it is a relative minimum by the second derivative test. Now find *y* and we'll be done.

$$y^2 = \frac{1}{2} + 2 = \frac{5}{2} \quad \rightarrow \quad y = \pm\frac{\sqrt{5}}{2} \quad \Rightarrow \quad \boxed{\left(\frac{1}{2}, \frac{\sqrt{5}}{2}\right),\ \left(\frac{1}{2}, -\frac{\sqrt{5}}{2}\right)}$$

10. $$\lim_{x\to -1} \frac{(x+1)^2}{\mathbf{e}^{2x+2} + 3x^2 + 4x} = \lim_{x\to -1} \frac{2(x+1)}{2\mathbf{e}^{2x+2} + 6x + 4} = \lim_{x\to -1} \frac{2}{4\mathbf{e}^{2x+2} + 6} = \frac{2}{10} = \boxed{\frac{1}{5}}$$

12. $$\lim_{x\to\infty}\left[x\sin\left(\frac{6}{x}\right)\right] = \lim_{x\to\infty}\frac{\sin\left(\frac{6}{x}\right)}{\frac{1}{x}} = \lim_{x\to\infty}\frac{-\frac{6}{x^2}\cos\left(\frac{6}{x}\right)}{-\frac{1}{x^2}} = \lim_{x\to\infty}\left[6\cos\left(\frac{6}{x}\right)\right] = 6\cos(0) = \boxed{6}$$

13. $$\boxed{du = 10t\sec^2\left(5t^2 - t\right)dt}$$

14. $\boxed{df = \left(2x\mathbf{e}^{9-x} - x^2\mathbf{e}^{9-x}\right)dx}$