

1. (2 pts) In this case we'll need to split the integral up do different substitutions on each one.

$$\begin{aligned}\int \sec^2(6x) + (1-x)^3 dx &= \int \sec^2(6x) dx + \int (1-x)^3 dx \\ u = 6x \quad du = 6dx \quad dx &= \frac{1}{6} du \quad v = 1-x \quad dv = -dx \quad dx = -dv \\ \int \sec^2(6x) + (1-x)^3 dx &= \frac{1}{6} \int \sec^2(u) du - \int v^3 dv \\ &= \frac{1}{6} \tan(u) - \frac{1}{4} v^4 + c = \boxed{\frac{1}{6} \tan(6x) - \frac{1}{4} (1-x)^4 + c}\end{aligned}$$

2. (2 pts) We'll first need to split this up and then use different substitutions on each one.

$$\begin{aligned}\int \frac{2-7z}{1+16z^2} dz &= \int \frac{2}{1+16z^2} dz - \int \frac{7z}{1+16z^2} dz = \int \frac{2}{1+(4z)^2} dz - \int \frac{7z}{1+16z^2} dz \\ u = 4z \quad du = 4dz \quad dz &= \frac{1}{4} du \quad v = 1+16z^2 \quad dv = 32z dz \quad z dz = \frac{1}{32} dv \\ \int \frac{2-7z}{1+16z^2} dz &= \frac{2}{4} \int \frac{1}{1+u^2} du - \frac{7}{32} \int \frac{1}{v} dv = \frac{1}{2} \tan^{-1}(u) - \frac{7}{32} \ln|v| + c \\ &= \boxed{\frac{1}{2} \tan^{-1}(4z) - \frac{7}{32} \ln|1+16z^2| + c}\end{aligned}$$

9. (2 pts) $\int_3^{-1} 12x^5 - 8x^3 + 1 dx = (2x^6 - 2x^4 + x) \Big|_3^{-1} = -1 - 1299 = \boxed{-1300}$

11. (2 pts) Because the "cut-off" point is in the middle of the interval of integration we'll need to split the integral up at the "cut-off" point and integrate each separately.

$$\begin{aligned}\int_{-3}^0 f(t) dt &= \int_{-3}^{-2} f(t) dt + \int_{-2}^0 f(t) dt = \int_{-3}^{-2} 1-t^2 dt + \int_{-2}^0 e^t dt \\ &= \left(t - \frac{1}{3}t^3\right) \Big|_{-3}^{-2} + e^t \Big|_{-2}^0 = \frac{2}{3} - (6) + 1 - e^{-2} = \boxed{-\frac{13}{3} - e^{-2} = -4.4686686}\end{aligned}$$

12. (2 pts) We first need to get rid of the absolute value bars. It's easy to see that the quantity in the absolute value bars will be zero at $w = 3$ and that we'll have $6 - 2w > 0$ if $w < 3$ and $6 - 2w < 0$ if $w > 3$. So,

$$\begin{aligned}\int_1^4 |6-2w| dw &= \int_1^3 |6-2w| dw + \int_3^4 |6-2w| dw = \int_1^3 6-2w dw - \int_3^4 6-2w dw \\ &= (6w - w^2) \Big|_1^3 - (6w - w^2) \Big|_3^4 = (9-5) - (8-9) = \boxed{5}\end{aligned}$$

Not Graded

3. $u = \ln(t) \quad du = \frac{1}{t} dt$

$$\int \frac{\cos(\ln(t)) - 2}{t} dt = \int \cos(u) - 2 du = \sin(u) - 2u + c = \boxed{\sin(\ln(t)) - 2\ln(t) + c}$$

4. This is one of those “tricky” substitution in the sense that you need to use it twice as follows.

$$\begin{aligned} u &= 2 + y^4 & du &= 4y^3 dy & y^3 dy &= \frac{1}{4} du & y^4 &= u - 2 \\ \int \frac{y^7}{2 + y^4} dy &= \int \frac{y^4 y^3}{2 + y^4} dy = \frac{1}{4} \int \frac{u - 2}{u} du = \frac{1}{4} \int 1 - \frac{2}{u} du = \frac{1}{4} (u - 2 \ln|u|) + c \\ &= \boxed{\frac{1}{4} (2 + y^4 - 2 \ln|2 + y^4|) + c} \end{aligned}$$

5. For this problem we have $\Delta x = \frac{1}{2}$ and the subintervals are [1, 1.5], [1.5, 2], [2, 2.5], [2.5, 3]. So, using the midpoints of each of these we get,

$$\text{Area} \approx \frac{1}{2} [g(1.25) + g(1.75) + g(2.25) + g(2.75)] = -10.17180965$$

For reference purposes the exact area is -10.40305447 (you don't need to worry about computing this, although if you take Calc II you should be able to compute it at that point).

6.

$$\begin{aligned} \int_{-3}^{10} 3g(x) - 12f(x) dx &= \int_{-3}^{10} 3g(x) dx - \int_{-3}^{10} 12f(x) dx = 3 \int_{-3}^{10} g(x) dx - 12 \int_{-3}^{10} f(x) dx \\ &= 3 \int_{-3}^{10} g(x) dx - 12 \left(- \int_{10}^{-3} f(x) dx \right) = 3(9) + 12(5) = \boxed{97} \end{aligned}$$

7.

$$\begin{aligned} \int_{-4}^{12} f(x) dx &= \int_{-4}^{20} f(x) dx + \int_{20}^8 f(x) dx + \int_8^{12} f(x) dx \\ &= \int_{-4}^{20} f(x) dx + \int_{20}^8 f(x) dx - \int_{12}^8 f(x) dx = 8 + (-3) - (1) = \boxed{4} \end{aligned}$$

$$8. \int_{x^2}^4 \ln(1 + e^{9t}) dt = - \int_4^{x^2} \ln(1 + e^{9t}) dt = \boxed{-2x \ln(1 + e^{9x^2})}$$

$$10. \int_0^{\frac{\pi}{4}} \cos(\alpha) - 3 \sin(\alpha) d\alpha = (\sin(\alpha) + 3 \cos(\alpha)) \Big|_0^{\frac{\pi}{4}} = \frac{\sqrt{2}}{2} + \frac{3\sqrt{2}}{2} - (0 + 3) = \boxed{2\sqrt{2} - 3}$$

13.

$$\begin{aligned} \int_2^3 5x^3 + 8x^{-3} - \frac{1}{4x} dx &= \left(\frac{5}{4} x^4 - 4x^{-2} - \frac{1}{4} \ln|x| \right) \Big|_2^3 = \frac{3629}{36} - \frac{1}{4} \ln(3) - \left(19 - \frac{1}{4} \ln(2) \right) \\ &= \boxed{\frac{2945}{36} + \frac{1}{4} \ln(3) - \frac{1}{4} \ln(2) = 81.7042} \end{aligned}$$

14. Now, before we jump right into this problem notice that we have division by zero issues in the second and third terms at $x = 0$ and because $x = 0$ is inside our interval of integration for this problem (as opposed to **#13** where it isn't in the interval) we **can't do this integral**. Note that the fact that we can integrate the first term is not relevant. We can't do the integral because of the problems in the second and third terms.