

1. (2 pts) Level Curve ($z = c$) : $c = 5x - y^2 \rightarrow x = \frac{1}{5}(c + y^2)$ So the level curve is a **parabola**.

Trace ($x = a$) : $z = 5a - y^2$ So this trace is a **parabola** in the yz -plane.

Trace ($y = b$) : $z = 5x - b^2$ So this trace is a **line** in the xz -plane.

5. (2 pts) $\vec{r}(\theta) = \langle 12 \cos \theta, 20, 12 \sin \theta \rangle$

6. (2 pts) $\vec{r}(t) = (1-t)\langle -7, 4, 2 \rangle + t\langle -3, -5, 1 \rangle$, $0 \leq t \leq 1$. Note that the range of t 's is important.

Without the range you get the line that goes through the two points and not the line segment starting at P and ending at Q .

8. (2 pts) For the first term we'll need to use L'Hospital's Rule. The rest can be done directly.

$$\begin{aligned}\lim_{t \rightarrow 2} \vec{r}(t) &= \lim_{t \rightarrow 2} \frac{\sin(t-2)}{t^2-4} \vec{i} + \lim_{t \rightarrow 2} e^{6-3t} \vec{j} - \lim_{t \rightarrow 2} (9t^2+8t-4) \vec{k} \\ &= \lim_{t \rightarrow 2} \frac{\cos(t-2)}{2t} \vec{i} + \lim_{t \rightarrow 2} e^{6-3t} \vec{j} - \lim_{t \rightarrow 2} (9t^2+8t-4) \vec{k} = \boxed{\frac{1}{4} \vec{i} + \vec{j} - 48 \vec{k}}\end{aligned}$$

10. (2 pts) Don't forget all the basic integration rules you learned in Calc I and Calc II. You will be asked to do those on occasion in this class. The first uses integration by parts and the third uses a simple trig identity. I'll leave the details to you to verify for each of these.

$$\begin{aligned}\int t \cos(t) dt &= t \sin(t) - \int \sin(t) dt = t \sin(t) + \cos(t) \\ \int e^{8t} dt &= \frac{1}{8} e^{8t} \\ \int \sin^2(4t) dt &= \int \frac{1}{2}(1 - \cos(8t)) dt = \frac{1}{2}(t - \frac{1}{8} \sin(8t)) \\ \boxed{\int \vec{r}(t) dt = \langle t \sin(t) + \cos(t), \frac{1}{8} e^{8t}, \frac{1}{2}(t - \frac{1}{8} \sin(8t)) \rangle + \vec{c}}\end{aligned}$$

Not Graded

2. Level Curve ($z = c$) : $2x^2 - 4y^2 + 2c^2 - 4 = 0$ So the level curve is a **hyperbola**.

Trace ($x = a$) : $-4y^2 + 2z^2 + 2a^2 - 4 = 0$ So this trace is a **hyperbola** in the yz -plane.

Trace ($y = b$) : $2x^2 + 2z^2 - 4b^2 - 4 = 0$ So this trace is a **circle** in the xz -plane.

3. $\vec{r}(x) = \langle x, e^{2x} - \sin(x+1) \rangle$

4. $\vec{r}(x, z) = \langle x, \cos(x+z) - z^2 + x^4, z \rangle$

7. (a) $\vec{r}_u \cdot \vec{r}_v = (2v)(0) + (-u^3)(uv) + (-1)(-8) = \boxed{8 - vu^4}$

(b) $\vec{r}_u \times \vec{r}_v = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 2v & -u^3 & -1 \\ 0 & uv & -8 \end{vmatrix} = 2v \begin{vmatrix} \vec{i} & \vec{j} \\ -u^3 & -1 \end{vmatrix} - uv \begin{vmatrix} \vec{i} & \vec{j} \\ 2v & -u^3 \end{vmatrix} - 8 \begin{vmatrix} \vec{i} & \vec{j} \\ 0 & uv \end{vmatrix}$
 $= 8u^3\vec{i} + 2uv^2\vec{k} - (-16v\vec{j}) - (-uv\vec{i}) = \boxed{(8u^3 + uv)\vec{i} + 16v\vec{j} + 2uv^2\vec{k}}$

(c) $\|\vec{r}_u \times \vec{r}_v\| = \sqrt{(8u^3 + uv)^2 + 256v^2 + 4u^2v^4}$

9. $\vec{r}'(t) = \left\langle \frac{1-t^2}{(t^2+1)^2}, \frac{6}{6t+1}, \cos(3t) - 3t \sin(3t) \right\rangle$

11. First we need,

$$\vec{r}'(t) = 2t e^{t^2-1} \vec{i} - 3 \vec{j} + [\sin(\pi t) + \pi t \cos(\pi t)] \vec{k} \quad \vec{r}(1) = \vec{i} - 2 \vec{j} \quad \vec{r}'(1) = 2\vec{i} - 3\vec{j} - \pi\vec{k}$$

The tangent line is then,

$$\boxed{\vec{r}(t) = \langle 1, -2, 0 \rangle + t \langle 2, -3, -\pi \rangle = \langle 1 + 2t, -2 - 3t, -\pi t \rangle}$$

12.

$$\vec{r}'(t) = \left\langle \frac{3}{t}, \sqrt{6}, t \right\rangle \quad \|\vec{r}'(t)\| = \sqrt{\frac{9}{t^2} + 6 + t^2} = \sqrt{\frac{9 + 6t^2 + t^4}{t^2}} = \sqrt{\frac{(t^2 + 3)^2}{t^2}} = \left| \frac{t^2 + 3}{t} \right| = \frac{t^2 + 3}{t}$$

You did recall that $\sqrt{a^2} = |a|$ right? Also, we can drop the absolute value bars because of the assumption that $t > 0$. The unit tangent is then,

$$T(t) = \frac{\vec{r}'(t)}{\|\vec{r}'(t)\|} = \frac{t}{t^2 + 3} \left\langle \frac{3}{t}, \sqrt{6}, t \right\rangle = \left\langle \frac{3}{t^2 + 3}, \frac{\sqrt{6}t}{t^2 + 3}, \frac{t^2}{t^2 + 3} \right\rangle$$

For the unit normal we have,

$$T'(t) = \left\langle \frac{-6t}{(t^2 + 3)^2}, \frac{\sqrt{6}(3 - t^2)}{(t^2 + 3)^2}, \frac{6t}{(t^2 + 3)^2} \right\rangle$$

$$\|T'(t)\| = \sqrt{\frac{36t^2}{(t^2 + 3)^4} + \frac{6(3 - t^2)^2}{(t^2 + 3)^4} + \frac{36t^2}{(t^2 + 3)^4}} = \sqrt{\frac{6t^4 + 36t^2 + 54}{(t^2 + 3)^4}} = \sqrt{\frac{6(t^2 + 3)^2}{(t^2 + 3)^4}} = \frac{\sqrt{6}}{t^2 + 3}$$

The unit normal is then,

$$\vec{N}(t) = \frac{T'(t)}{\|T'(t)\|} = \frac{t^2 + 3}{\sqrt{6}} \left\langle \frac{-6t}{(t^2 + 3)^2}, \frac{\sqrt{6}(3-t^2)}{(t^2 + 3)^2}, \frac{6t}{(t^2 + 3)^2} \right\rangle = \boxed{\left\langle \frac{-\sqrt{6}t}{t^2 + 3}, \frac{(3-t^2)}{t^2 + 3}, \frac{\sqrt{6}t}{t^2 + 3} \right\rangle}$$