

4. (2 pts) This function is not continuous at $(0,0)$ so let's see if we can find a couple of paths that give different values for the limit.

$$\text{y-axis } (x=0) : \lim_{(x,y) \rightarrow (0,0)} \frac{x^4 y^2}{2x^{12} + 6y^3} = \lim_{(0,y) \rightarrow (0,0)} \frac{0}{6y^3} = 0$$

$$y = x^4 : \lim_{(x,y) \rightarrow (0,0)} \frac{x^4 y^2}{2x^{12} + 6y^3} = \lim_{(x,x^4) \rightarrow (0,0)} \frac{x^4(x^8)}{2x^{12} + 6(x^{12})} = \lim_{(x,x^4) \rightarrow (0,0)} \frac{x^{12}}{8x^{12}} = \lim_{(x,x^4) \rightarrow (0,0)} \frac{1}{8} = \frac{1}{8}$$

So, we have two paths that give different values of the limit and so we know that $\lim_{(x,y) \rightarrow (0,0)} \frac{x^4 y^2}{2x^{12} + 6y^3}$ doesn't exist.

6. (2 pts)

$$\frac{\partial w}{\partial x} = y \cos(xy) e^{2y+z^2} \quad \frac{\partial w}{\partial z} = 2z \sin(xy) e^{2y+z^2}$$

$$\frac{\partial w}{\partial y} = x \cos(xy) e^{2y+z^2} + 2 \sin(xy) e^{2y+z^2}$$

$$7. \quad f_u = 2u \ln(s^2 - 8t^4) + 4 \sec^2(4u) \quad f_v = 0 \quad f_s = \frac{2su^2}{s^2 - 8t^4} \quad f_t = \frac{-32t^3u^2}{s^2 - 8t^4}$$

9. (2 pts)

$$z_x = \frac{1}{x} + 6y^2x^5 \quad z_y = -\frac{1}{y} + 2yx^6 - 4$$

$$z_{xx} = -\frac{1}{x^2} + 30y^2x^4 \quad z_{xy} = 12yx^5 \quad z_{yx} = 12yx^5 \quad z_{yy} = \frac{1}{y^2} + 2x^6$$

11. (2 pts) The key here is to recall that they can be done in any order we to make life simpler do the s derivative followed by 4 t derivatives.

$$f_s = -\frac{t^6}{\sqrt{1-2s}} \quad f_{st} = -\frac{6t^5}{\sqrt{1-2s}} \quad f_{stt} = -\frac{30t^4}{\sqrt{1-2s}} \quad f_{sttt} = -\frac{120t^3}{\sqrt{1-2s}} \quad \boxed{f_{ttstt} = f_{stttt} = -\frac{360t^2}{\sqrt{1-2s}}}$$

Not Graded

$$1. \quad L = \int_1^{10} \|r'(t)\| dt = \int_1^{10} \frac{t^2 + 3}{t} dt = \left(\frac{1}{2}t^2 + 3 \ln(t) \right) \Big|_1^{10} = \boxed{\left[\frac{99}{2} + 3 \ln(10) \right]}$$

2. Not much to do here. This function is continuous at $(0,0)$ so : $\lim_{(x,y) \rightarrow (0,0)} \frac{\cos(x) - \sin(y)}{xy + x^2 - y^2 + 4} = \boxed{\frac{1}{4}}$

3. This function is not continuous at $(0,0)$ so let's see if we can find a couple of paths that give different values for the limit. I'll leave it to you to verify that both the x -axis and y -axis give the same value and so we can only use one of them. I'll use the x -axis.

$$\text{x-axis } (y=0) : \lim_{(x,y) \rightarrow (0,0)} \frac{(7x-4y)^3}{2y^3+x^3} = \lim_{(x,0) \rightarrow (0,0)} \frac{(7x)^3}{x^3} = \lim_{(x,0) \rightarrow (0,0)} 343 = \underline{\underline{343}}$$

$$y=x : \lim_{(x,y) \rightarrow (0,0)} \frac{(7x-4y)^3}{2y^3+x^3} = \lim_{(x,x) \rightarrow (0,0)} \frac{(3x)^3}{3x^3} = \lim_{(x,x) \rightarrow (0,0)} 9 = \underline{\underline{9}}$$

So, we have two paths that give different values of the limit and so we know that $\lim_{(x,y) \rightarrow (0,0)} \frac{(7x-4y)^3}{2y^3+x^3}$ doesn't exist.

$$5. \quad g_x = -\frac{4y^3z^2}{x^5} + \frac{1}{\sqrt{4z+2x}} \quad g_y = \frac{3y^2z^2}{x^4} - 21\cos^2(7y)\sin(7y) \quad g_z = \frac{2y^3z}{x^4} + \frac{2}{\sqrt{4z+2x}}$$

8.

$$4y^2z^3 \frac{\partial z}{\partial x} - \sec^2(1-x) = 18z^5 \frac{\partial z}{\partial x} \quad \Rightarrow \quad \boxed{\frac{\partial z}{\partial x} = \frac{\sec^2(1-x)}{4y^2z^3 - 18z^5}}$$

$$2yz^4 + 4y^2z^3 \frac{\partial z}{\partial y} = 18z^5 \frac{\partial z}{\partial y} \quad \Rightarrow \quad \boxed{\frac{\partial z}{\partial y} = \frac{2yz^4}{18z^5 - 4y^2z^3}}$$

10.

$$h_s = -4s^3t^3 \sin(t^3s^4) + \frac{4s^3}{t^2} \quad h_t = -3t^2s^4 \sin(t^3s^4) - \frac{2s^4}{t^3}$$

$$h_{ss} = -12s^2t^3 \sin(t^3s^4) - 16s^6t^6 \cos(t^3s^4) + \frac{12s^2}{t^2}$$

$$h_{st} = -12s^3t^2 \sin(t^3s^4) - 12s^7t^5 \cos(t^3s^4) - \frac{8s^3}{t^3}$$

$$h_{ts} = -12t^2s^3 \sin(t^3s^4) - 12t^5s^7 \cos(t^3s^4) - \frac{8s^3}{t^3}$$

$$h_{tt} = -6ts^4 \sin(t^3s^4) - 9t^4s^8 \sin(t^3s^4) + \frac{6s^4}{t^4}$$

12. In this case we don't need to worry so much about order as we did with #11 as none of the terms will drop out so we'll just go in the given order.

$$\frac{\partial u}{\partial z} = -\frac{x^4 y^{\frac{3}{2}}}{z^2} + 7x^3 z^6 y^2 \quad \frac{\partial^2 u}{\partial z^2} = \frac{2x^4 y^{\frac{3}{2}}}{z^3} + 42x^3 z^5 y^2 \quad \frac{\partial^3 u}{\partial y \partial z^2} = \frac{3x^4 y^{\frac{1}{2}}}{z^3} + 84x^3 z^5 y$$
$$\frac{\partial^4 u}{\partial x \partial y \partial z^2} = \frac{12x^3 y^{\frac{1}{2}}}{z^3} + 252x^2 z^5 y \quad \frac{\partial^5 u}{\partial x^2 \partial y \partial z^2} = \frac{36x^2 y^{\frac{1}{2}}}{z^3} + 504xz^5 y$$
$$\boxed{\frac{\partial^6 u}{\partial y \partial x^2 \partial y \partial z^2} = \frac{18x^2 y^{-\frac{1}{2}}}{z^3} + 504xz^5}$$