Math 2415 **Set 3** **10 Points**

Chain Rule

1. Use the Chain Rule to find $\frac{dw}{dt}$ given that,

$$w = \ln(2x + 4z) + y^2 x^3 \qquad x = \frac{1}{t^5} \quad y = t^3 \quad z = \mathbf{e}^{2t}$$

2. Use the Chain Rule to find $\frac{dz}{dy}$ given that $z = y^2 \cos(1 + x^2)$, $x = 8 - y^3$

3. Use the Chain Rule to find $\frac{\partial w}{\partial s}$ and $\frac{\partial w}{\partial t}$ given that,

$$w = \sin(yx^2) - y^4 + 2x, \qquad x = 3t - 8s \quad y = p^2 \quad p = t^5$$

4. Write down the Chain Rule to find $\frac{\partial w}{\partial t}$ for the following situation.

$$w = f(x, y, z) \quad x = x(s,t) \quad y = y(t), \quad z = z(s, p) \quad p = p(u,v) \quad v = v(t)$$

5. Use the Chain Rule to find $\frac{\partial z}{\partial x}$ and $\frac{\partial z}{\partial y}$ for $y^2 z^4 + \tan(1 - x) = 3z^6 + 1$.

Directional Derivatives

6. Find ∇f and the directional derivative for $f(x, y) = \sin(9x - y^2)$ in the direction of $\vec{v} = \langle -6, 1 \rangle$ at the point $(1, -3)$.

7. Find the directional derivative of $f(x, y, z) = z\mathbf{e}^{x^2 - z} + y$ in the direction of $\vec{v} = \langle 1, 4, -3 \rangle$.

8. Find the maximum rate of change of $f(x, y, z) = x^2 - y^4 z + \frac{y - z}{x}$ at the point $(1, 2, -1)$ and the direction in which it occurs.

9. Given that $\vec{u} = \left\langle -\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}} \right\rangle$, $\vec{v} = \left\langle \frac{2}{\sqrt{20}}, \frac{-4}{\sqrt{20}} \right\rangle$, $\vec{w} = \left\langle -\frac{4}{5}, \frac{3}{5} \right\rangle$, $D_{\vec{u}} f(0, -3) = -\frac{5}{\sqrt{2}}$ and $D_{\vec{v}} f(0, -3) = \frac{6}{\sqrt{5}}$ determine the value of $D_{\vec{w}} f(0, -3)$.

Tangent Planes

10. Find the equation of the tangent plane to $z = x^2 \cos(5x - y) - 2y$ at $(1, 5)$.