

1. (2 pts)

$$\begin{aligned}\frac{dw}{dt} &= \frac{dw}{dx} \frac{dx}{dt} + \frac{dw}{dy} \frac{dy}{dt} + \frac{dw}{dz} \frac{dz}{dt} \\ &= \left[\left(\frac{2}{2x+4z} + 3y^2 x^2 \right) (-5t^{-6}) + 2yx^3 (3t^2) + \frac{4}{2x+4z} (2e^{2t}) \right] \\ &= \boxed{-5t^{-6} \left(\frac{2}{2t^{-5} + 4e^{2t}} + 3t^{-4} \right) + 6t^{-10} + \frac{8e^{2t}}{2t^{-5} + 4e^{2t}}}\end{aligned}$$

4. (3 pts) $\boxed{\frac{\partial w}{\partial t} = \frac{\partial w}{\partial x} \frac{\partial x}{\partial t} + \frac{\partial w}{\partial y} \frac{\partial y}{\partial t} + \frac{\partial w}{\partial z} \frac{\partial z}{\partial p} \frac{\partial p}{\partial v} \frac{\partial v}{\partial t}}$

6. (2 pts) Don't forget that the vector needs to be a unit vector.

$$\begin{aligned}\nabla f(x, y) &= \langle 9 \cos(9x - y^2), -2y \cos(9x - y^2) \rangle \quad \nabla f(1, -3) = \langle 9, 6 \rangle \quad \vec{u} = \left\langle \frac{-6}{\sqrt{37}}, \frac{1}{\sqrt{37}} \right\rangle \\ D_{\vec{u}} f(1, -3) &= \langle 9, 6 \rangle \cdot \left\langle \frac{-6}{\sqrt{37}}, \frac{1}{\sqrt{37}} \right\rangle = \frac{-54}{\sqrt{37}} + \frac{6}{\sqrt{37}} = \boxed{\frac{-48}{\sqrt{37}}}\end{aligned}$$

9. (3 pts) Notice that all the vectors are unit vectors. Now all we need to do is set up the equations,

$$\begin{aligned}D_{\vec{u}} f(0, -3) &= -\frac{1}{\sqrt{2}} f_x(0, -3) + \frac{1}{\sqrt{2}} f_y(0, -3) = -\frac{5}{\sqrt{2}} \\ D_{\vec{v}} f(0, -3) &= \frac{2}{\sqrt{20}} f_x(0, -3) - \frac{4}{\sqrt{20}} f_y(0, -3) = \frac{6}{\sqrt{5}}\end{aligned}$$

This is a system of equations that we can solve for the “unknowns” $f_x(0, -3)$ and $f_y(0, -3)$. Doing so gives $f_x(0, -3) = 4$ and $f_y(0, -3) = -1$. We can now compute,

$$D_{\vec{w}} f(0, -3) = -\frac{4}{5}(4) + \frac{3}{5}(-1) = \boxed{-\frac{19}{5}}$$

Not Graded**2.**

$$\begin{aligned}\frac{dz}{dy} &= \frac{\partial z}{\partial x} \frac{dx}{dy} + \frac{\partial z}{\partial y} = -2xy^2 \sin(1+x^2)(-3y^2) + 2y \cos(1+x^2) \\ &= \boxed{6xy^4 \sin(1+x^2) + 2y \cos(1+x^2)}\end{aligned}$$

3.

$$\begin{aligned}\frac{\partial w}{\partial s} &= \frac{\partial w}{\partial x} \frac{\partial x}{\partial s} + \frac{\partial w}{\partial y} \frac{\partial y}{\partial p} \frac{\partial p}{\partial s} \\ &= (2xy \cos(yx^2) + 2)(-8) + (x^2 \cos(yx^2) - 4y^3)(2p)(0) = \boxed{-16(xy \cos(yx^2) + 1)}\end{aligned}$$

$$\begin{aligned}\frac{\partial w}{\partial t} &= \frac{\partial w}{\partial x} \frac{\partial x}{\partial t} + \frac{\partial w}{\partial y} \frac{\partial y}{\partial p} \frac{\partial p}{\partial t} \\ &= (2xy \cos(yx^2) + 2)(3) + (x^2 \cos(yx^2) - 4y^3)(2p)(5t^4) \\ &= \boxed{6(xy \cos(yx^2) + 1) + 10pt^4(x^2 \cos(yx^2) - 4y^3)}\end{aligned}$$

5. Note that your answers should match #8 on the previous homework set....

$$y^2 z^4 + \tan(1-x) - 3z^6 - 1 = 0$$

$$\frac{\partial z}{\partial x} = -\frac{-\sec^2(1-x)}{4y^2 z^3 - 18z^5} = \frac{\sec^2(1-x)}{4y^2 z^3 - 18z^5}$$

$$\frac{\partial z}{\partial y} = -\frac{2yz^4}{4y^2 z^3 - 18z^5} = \frac{2yz^4}{18z^5 - 4y^2 z^3}$$

7.

$$\nabla f(x, y, z) = \left\langle 2xz e^{x^2-z}, 1, e^{x^2-z} - z e^{x^2-z} \right\rangle \quad \vec{u} = \left\langle \frac{1}{\sqrt{26}}, \frac{4}{\sqrt{26}}, \frac{-3}{\sqrt{26}} \right\rangle$$

$$D_{\vec{u}} f(x, y, z) = \boxed{\frac{1}{\sqrt{26}} \left(2xz e^{x^2-z} + 4 - 3(e^{x^2-z} - z e^{x^2-z}) \right)}$$

8. We'll need the gradient for this.

$$\nabla f(x, y, z) = \left\langle 2x - \frac{y-z}{x^2}, -4y^3 z + \frac{1}{x}, -y^4 - \frac{1}{x} \right\rangle$$

$$\nabla f(1, 2, -1) = \langle -1, 33, -17 \rangle \quad \|\nabla f(1, 2, -1)\| = \sqrt{1379}$$

So, the maximum rate of change of the function is $\sqrt{1379}$ and it occurs in the direction of $\langle -1, 33, -17 \rangle$.

10.

$$\begin{array}{ll} f(x, y) = x^2 \cos(5x - y) - 2y & f(1, 5) = -9 \\ f_x(x, y) = 2x \cos(5x - y) - 5x^2 \sin(5x - y) & f_x(1, 5) = 2 \\ f_y(x, y) = x^2 \sin(5x - y) - 2 & f_y(1, 5) = -2 \end{array}$$

The tangent line is then,

$$z = -9 + (2)(x-1) + (-2)(y-5) = \boxed{-9 + 2(x-1) - 2(y-5)}$$