

1. (2 pts)

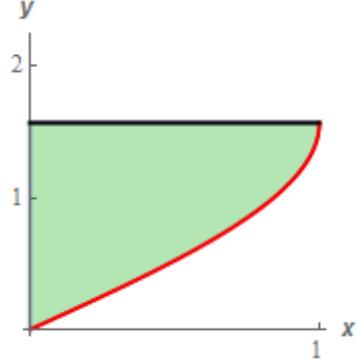
$$\begin{aligned} \int_1^{-3} \int_0^2 x^2 y^7 \cos(x^3 y^4) dx dy &= \int_1^{-3} \left(\frac{1}{3} y^3 \sin(x^3 y^4) \right) \Big|_0^2 dy = \int_1^{-3} \frac{1}{3} y^3 \sin(8y^4) dy \\ &= \left(-\frac{1}{96} \cos(8y^4) \right) \Big|_1^{-3} = \boxed{\frac{1}{96} (\cos(8) - \cos(648)) = -0.00853} \end{aligned}$$

3. (2 pts) $\iint_R y e^{3y-x} dA = \int_1^2 \int_{-1}^0 y e^{3y-x} dy dx$. We'll need to use integration by parts for the first integral.

$$\begin{aligned} u &= y & du &= dy & dv &= e^{3y-x} dy & v &= \frac{1}{3} e^{3y-x} \\ \iint_R y e^{3y-x} dA &= \int_1^2 \left(\frac{1}{3} y e^{3y-x} - \frac{1}{3} \int e^{3y-x} dy \right) \Big|_{-1}^0 dx = \int_1^2 \left(\frac{1}{3} y e^{3y-x} - \frac{1}{9} e^{3y-x} \right) \Big|_{-1}^0 dx \\ &= \int_1^2 \frac{4}{9} e^{-3x} - \frac{1}{9} e^{-x} dx = \left(-\frac{4}{9} e^{-3x} + \frac{1}{9} e^{-x} \right) \Big|_1^2 = \boxed{-\frac{4}{9} e^{-5} + \frac{1}{9} e^{-2} + \frac{4}{9} e^{-4} - \frac{1}{9} e^{-1} = -0.0207} \end{aligned}$$

6. (2 pts) A sketch is to the right. Limits: $0 \leq y \leq \frac{1}{2}\pi$, $0 \leq x \leq \sin(y)$

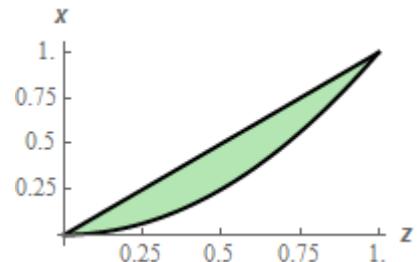
$$\begin{aligned} \iint_D \sqrt[3]{1 - \cos(y)} dA &= \int_0^{\frac{1}{2}\pi} \int_0^{\sin(y)} (1 - \cos(y))^{\frac{1}{3}} dx dy \\ &= \int_0^{\frac{1}{2}\pi} x (1 - \cos(y))^{\frac{1}{3}} \Big|_0^{\sin(y)} dy \\ &= \int_0^{\frac{1}{2}\pi} \sin(y) (1 - \cos(y))^{\frac{1}{3}} dy = \frac{3}{4} (1 - \cos(y))^{\frac{4}{3}} \Big|_0^{\frac{1}{2}\pi} \\ &= \boxed{\frac{3}{4}} \end{aligned}$$

**8. (2 pts)** The formula here is,

$$V = \iint_D f(x, z) dA \quad D \text{ is in the } xz\text{-plane}$$

The region (note the orientation of the axes!) is shown to the right and the limits are,

$$0 \leq x \leq 1, \quad x^2 \leq z \leq x$$



The volume is,

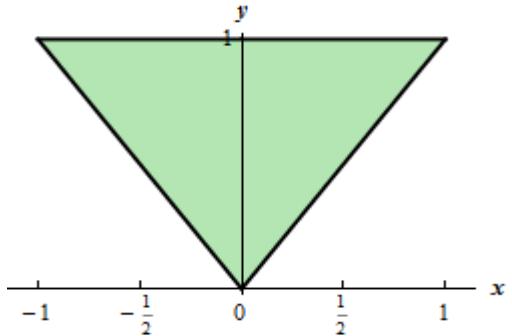
$$\begin{aligned} V &= \iint_D 8 - 2x^2 - 2z^2 dA = \int_0^1 \int_{x^2}^x (8 - 2x^2 - 2z^2) dz dx = \int_0^1 \left(8z - 2x^2 z - \frac{2}{3} z^3 \right) \Big|_{x^2}^x dx \\ &= \int_0^1 \left(8x - 8x^2 - \frac{8}{3} x^3 + 2x^4 + \frac{2}{3} x^6 \right) dx = \left(4x^2 - \frac{8}{3} x^3 - \frac{2}{3} x^4 + \frac{2}{5} x^5 + \frac{2}{21} x^7 \right) \Big|_0^1 = \boxed{\frac{122}{105} = 1.1619} \end{aligned}$$

9. (2 pts) From the limits of the integral we can see that the limits on the variables are : $0 \leq y \leq 1$, $-y \leq x \leq y$. A sketch of this region is to the right.

Interchanging limits will require us to do two integrals. The limits for each are,

$$\begin{array}{ll} -1 \leq x \leq 0 & 0 \leq x \leq 1 \\ -x \leq y \leq 1 & x \leq y \leq 1 \end{array}$$

The integral is then,



$$\begin{aligned} \int_0^1 \int_{-y}^y 8yx^3 \, dx \, dy &= \int_{-1}^0 \int_{-x}^1 8yx^3 \, dy \, dx + \int_0^1 \int_x^1 8yx^3 \, dy \, dx = \int_{-1}^0 4y^2x^3 \Big|_{-x}^1 \, dx + \int_0^1 4y^2x^3 \Big|_x^1 \, dx \\ &= \int_{-1}^0 4x^3 - 4x^5 \, dx + \int_0^1 4x^3 - 4x^5 \, dx = \left(x^4 - \frac{2}{3}x^6 \right) \Big|_{-1}^0 + \left(x^4 - \frac{2}{3}x^6 \right) \Big|_0^1 = -\frac{1}{3} + \frac{1}{3} = \boxed{0} \end{aligned}$$

Not Graded

2.

$$\begin{aligned} \iint_R x^2 \cos^2 \left(\frac{y}{2} \right) + \frac{4x^3}{x^4 + 1} \, dA &= \int_0^4 \int_{-1}^2 x^2 \cos^2 \left(\frac{y}{2} \right) + \frac{4x^3}{x^4 + 1} \, dx \, dy = \int_0^4 \left(\frac{1}{3}x^3 \cos^2 \left(\frac{y}{2} \right) + \ln|x^4 + 1| \right) \Big|_{-1}^2 \, dy \\ &= \int_0^4 \frac{2}{3} \cos^2 \left(\frac{y}{2} \right) + \ln(17) - \ln(2) \, dy = \int_0^4 \frac{1}{3}(1 + \cos(y)) + \ln(\frac{17}{2}) \, dy \\ &= \left(\frac{1}{3}(y + \sin(y)) + \ln(\frac{17}{2})y \right) \Big|_0^4 = \boxed{\left[\frac{1}{3}(4 + \sin(4)) + 4 \ln(\frac{17}{2}) \right]} \end{aligned}$$

4.

$$\begin{aligned} \int_0^1 \int_{\sqrt{y}}^{2+y} x^3 + \frac{1}{\sqrt{y}} - 4 \, dx \, dy &= \int_0^1 \left(\frac{1}{4}x^4 + \frac{x}{\sqrt{y}} - 4x \right) \Big|_{\sqrt{y}}^{2+y} \, dy \\ &= \int_0^1 \frac{1}{4}(2+y)^4 + \frac{2+y}{\sqrt{y}} - 4(2+y) - \left(\frac{1}{4}y^2 + 1 - 4\sqrt{y} \right) \, dy \\ &= \int_0^1 \frac{1}{4}(2+y)^4 + 2y^{-\frac{1}{2}} + 5y^{\frac{1}{2}} - 9 - 4y - \frac{1}{4}y^2 \, dy \\ &= \left(\frac{1}{20}(2+y)^5 + 4y^{\frac{1}{2}} + \frac{10}{3}y^{\frac{3}{2}} - 9y - 2y^2 - \frac{1}{12}y^3 \right) \Big|_0^1 = \boxed{\left[\frac{34}{5} \right]} \end{aligned}$$

5.

$$\begin{aligned} \iint_D \frac{e^{x^4+1}}{\sqrt{y}} dA &= \int_0^1 \int_0^{x^6} \frac{e^{x^4+1}}{\sqrt{y}} dy dx = \int_0^1 \left(2y^{\frac{1}{2}} e^{x^4+1} \right) \Big|_0^{x^6} dx \\ &= \int_0^1 2x^3 e^{x^4+1} dx = \left(\frac{1}{2} e^{x^4+1} \right) \Big|_0^1 = \boxed{\left[\frac{1}{2} [e^2 - e^1] \right] = 2.3354} \end{aligned}$$

7. A sketch is to the right. Here are the limits for each part.

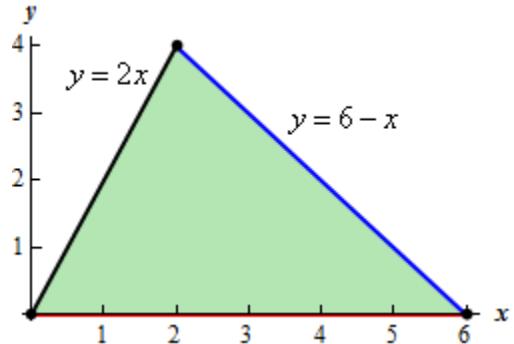
(a) $0 \leq x \leq 2, 0 \leq y \leq 2x$ AND $2 \leq x \leq 6, 0 \leq y \leq 6-x$

In this case we'll need two integrals b/c the upper function changes at $x = 2$.

(b) $0 \leq y \leq 4, \frac{1}{2}y \leq x \leq 6-y$

Here don't forget to write the equations in the form

$x = f(y)$ for the x limits.



The integrals are then,

(a)

$$\begin{aligned} \iint_D 12y dA &= \iint_{D_1} 12y dA + \iint_{D_2} 12y dA = \int_0^2 \int_0^{2x} 12y dy dx + \int_2^6 \int_0^{6-x} 12y dy dx \\ &= \int_0^2 6y^2 \Big|_0^{2x} dx + \int_2^6 6y^2 \Big|_0^{6-x} dx = \int_0^2 24x^2 dx + \int_2^6 6(6-x)^2 dx \\ &= 8x^3 \Big|_0^2 - 2(6-x)^3 \Big|_2^6 = 64 + 128 = \boxed{192} \end{aligned}$$

(b)

$$\iint_D 12y dA = \int_0^4 \int_{\frac{1}{2}y}^{6-y} 12y dx dy = \int_0^4 12yx \Big|_{\frac{1}{2}y}^{6-y} dy = \int_0^4 72y - 18y^2 dy = \left(36y^2 - 6y^3 \right) \Big|_0^4 = \boxed{192}$$

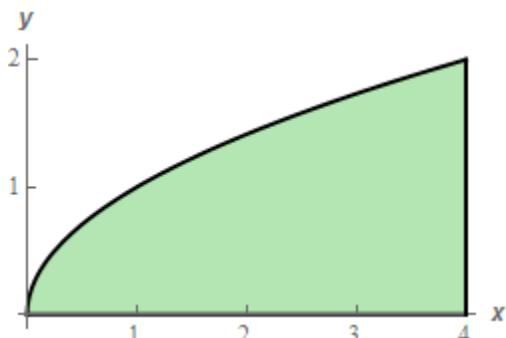
10. From the limits of the integral we can see that the limits

on the variables are : $0 \leq y \leq 2, y^2 \leq x \leq 4$. A sketch of this region is to the right.

Interchanging limits will give us the following limits,

$$0 \leq x \leq 4, 0 \leq y \leq \sqrt{x}$$

The integral is then,



$$\begin{aligned} \int_0^2 \int_{y^2}^4 y^7 e^{2+x^5} dx dy &= \int_0^4 \int_0^{\sqrt{x}} y^7 e^{2+x^5} dy dx = \int_0^4 \frac{1}{8} y^8 e^{2+x^5} \Big|_0^{\sqrt{x}} dx \\ &= \int_0^4 \frac{1}{8} y^4 e^{2+x^5} dx = \left(\frac{1}{40} e^{2+x^5} \right) \Big|_0^4 = \boxed{\left[\frac{1}{40} (e^{1026} - e^2) \right]} \end{aligned}$$

11. We'll clearly need two integrals to do this problem. Here are the limits for each of them,

$$-2 \leq x \leq 2, \quad x^2 \leq y \leq 4 \quad -4 \leq y \leq 0, \quad (y+1)^2 \leq x \leq 4$$

The integral is then,

$$\begin{aligned} \iint_D 4x+1 \, dA &= \iint_{D_1} 4x+1 \, dA + \iint_{D_2} 4x+1 \, dA = \int_{-2}^2 \int_{x^2}^4 4x+1 \, dy \, dx + \int_{-4}^0 \int_{(y+1)^2}^4 4x+1 \, dx \, dy \\ &= \int_{-2}^2 (4x+1) y \Big|_{x^2}^4 \, dx + \int_{-4}^0 (2x^2+x) \Big|_{(y+1)^2}^4 \, dy \\ &= \int_{-2}^2 16x+4-4x^3-x^2 \, dx + \int_{-4}^0 36-2(y+1)^4-(y+1)^2 \, dy \\ &= \left(8x^2+4x-x^4-\frac{1}{3}x^3\right) \Big|_{-2}^2 + \left(36y-\frac{2}{5}(y+1)^5-\frac{1}{3}(y+1)^3\right) \Big|_{-4}^0 = \frac{32}{3} + \frac{556}{15} = \boxed{\frac{716}{15}} \end{aligned}$$